Nonlinear Energy Harvesting

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&

Wisepower

N.i.P.S Laboratory

Noise in Physical Systems

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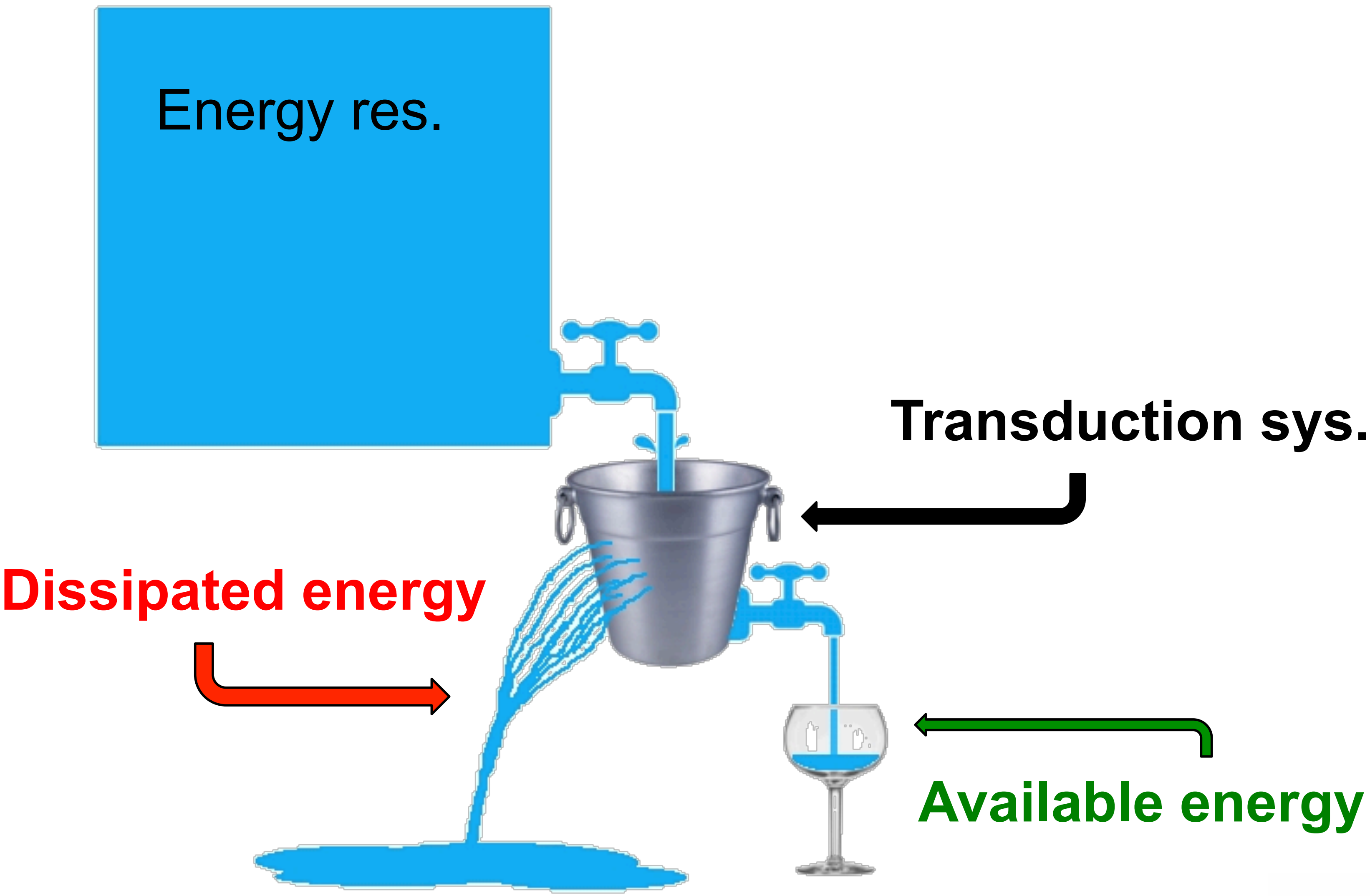
Energy harvesting basic ideas



Power Spectrum

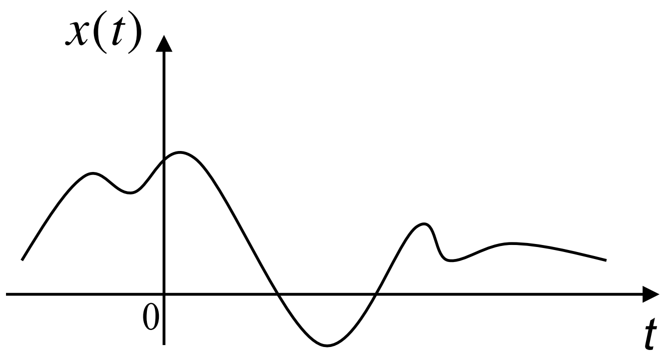
For a deterministic signal *x*(t), the spectrum is well defined: If $X(f)$ represents its Fourier transform, i.e., if

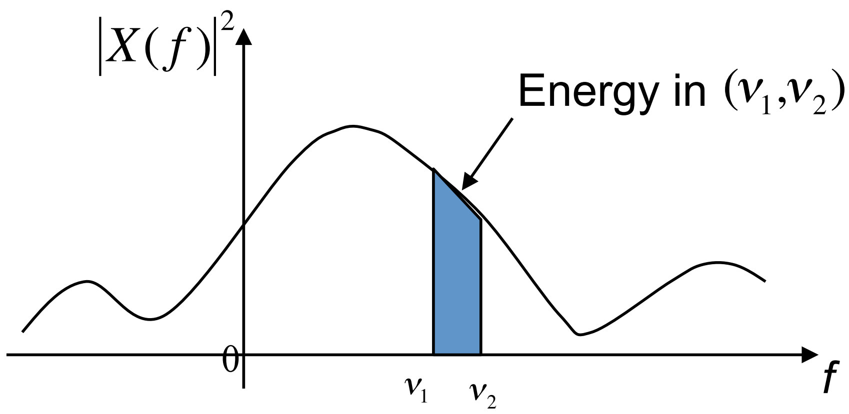
$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-i2\pi f} dt$$

then $|X(f)|^2$ represents its energy spectrum. This follows from Parseval's theorem since the signal energy is given by

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$$

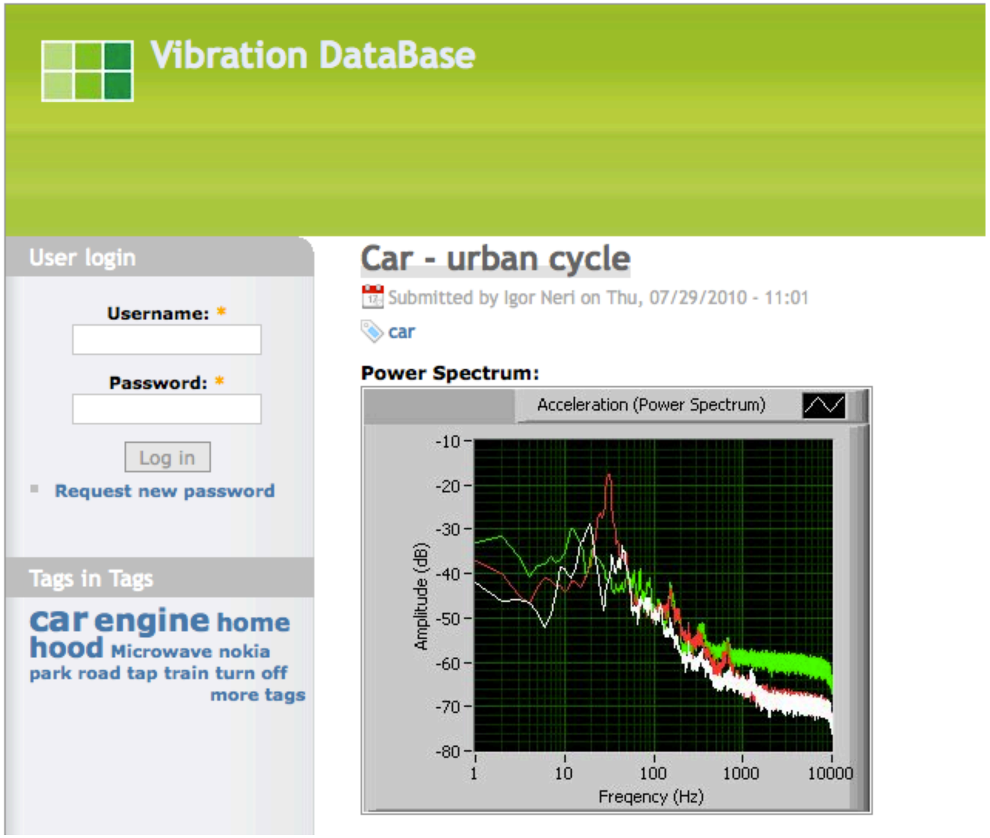
Thus $\int_{\nu_1}^{\nu_2} |X(f)|^2 df$ represents the signal energy in the band (ν_1, ν_2)





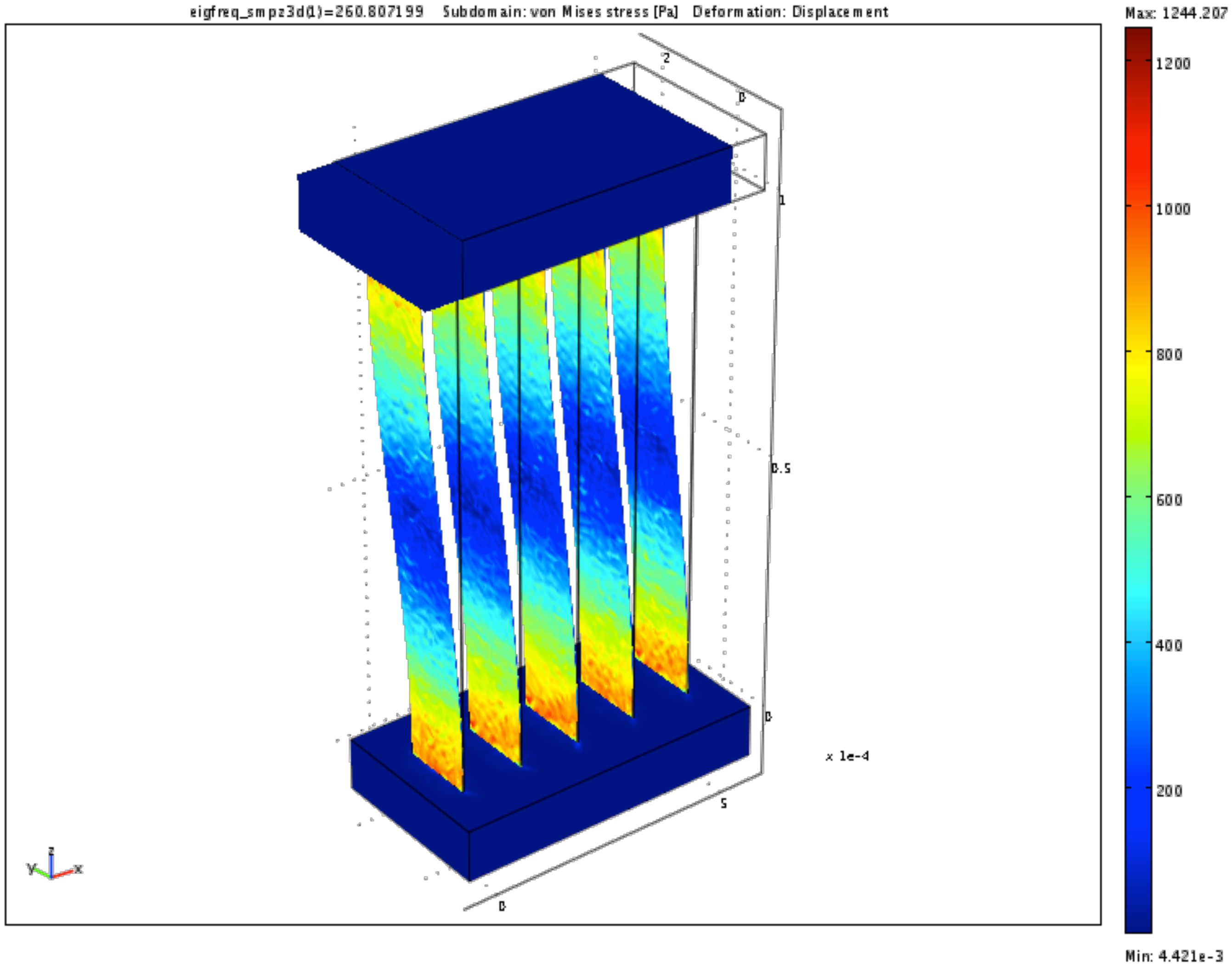
Noise spectrum examples

You'll see some noise spectrum in our database…
(See Neri's presentation at the workshop the 7th)



Linear system

Micro energy harvesting system…



25 μm thick
1 mm high

Freq. 10 KHz

Noise energy harvesting

Wish list for the perfect vibration harvester

1) Capable of harvesting energy on a broad-band
2) No need for frequency tuning
3) Capable of harvesting energy at low frequency

1) Non-resonant system
2) “Transfer function” with wide frequency resp.
3) Low frequency operated

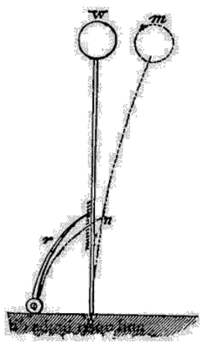
Noise energy harvesting

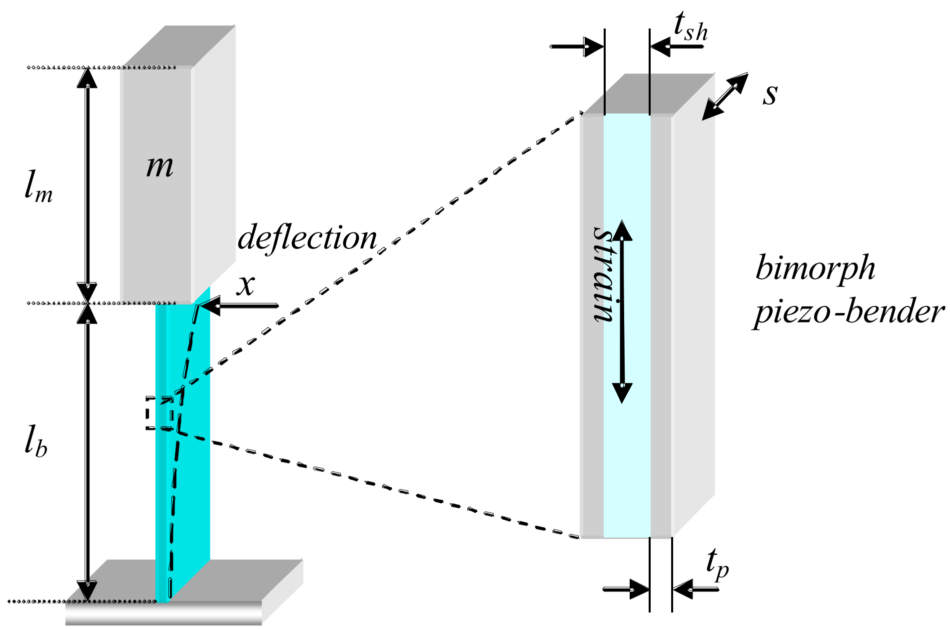
NON-Linear mechanical oscillators

1) Non-resonant system
2) “Transfer function” with wide frequency resp.
3) Low frequency operated

Example…

Inverted pendulum





F. Cottone, PhD Thesys, Perugia 2007

Statistics

- "1D" Statistics: (2nd Order Cumulants, 1st Order Spectra)
  - Correlation: $C_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, d\tau \quad \Leftrightarrow \quad X(f)\, Y^*(f) = S_{xy}(f)$
  - Power Spectral Density: $C_{2x}(t) \quad \Leftrightarrow \quad X(f)\, X^*(f) = S_{2x}(f)$
  - Coherence: $\mathrm{C}_{xy}(f) = \dfrac{S_{xy}(f)}{\sqrt{S_{2x}(f)\, S_{2y}(f)}}$
    - Tells us power and phase coherence at a given frequency

Statistics

- "2D" Statistics: (3rd Order Cumulants, 2nd Order Spectra)

  – Bicumulant:

$$C_{xyz}(t,t') = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, z(t'+\tau)\, d\tau \quad \Leftrightarrow \quad X(f_1)\, Y(f_2)\, Z^*(f_1+f_2) = S_{xyz}(f_1,f_2)$$

  – Bispectral Density: $C_{3x}(t) \quad \Leftrightarrow \quad X(f_1)\, X(f_2)\, X^*(f_1+f_2) = S_{3x}(f_1,f_2)$

$$S_{3x}(f_1,f_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_{3x}(m,n)\, e^{2\pi i(f_1 m + f_2 n)}\, dm\, dn$$

  – Bicoherence: $\mathbf{c}_{xyz}(f) = \dfrac{S_{xyz}(f_1,f_2)}{\sqrt{S_{xx}(f_1)}\sqrt{S_{yy}(f_2)}\sqrt{S_{zz}(f_1+f_2)}}$

    - Tells us power and phase coherence at a coupled frequency

Statistics

The Spectrogram (*STFT square modulus*):

$$S_x(t,\nu) = \left| \int_{-\infty}^{+\infty} x(\tau) h^*(\tau - t) e^{-i2\pi\nu\tau} d\tau \right|^2$$

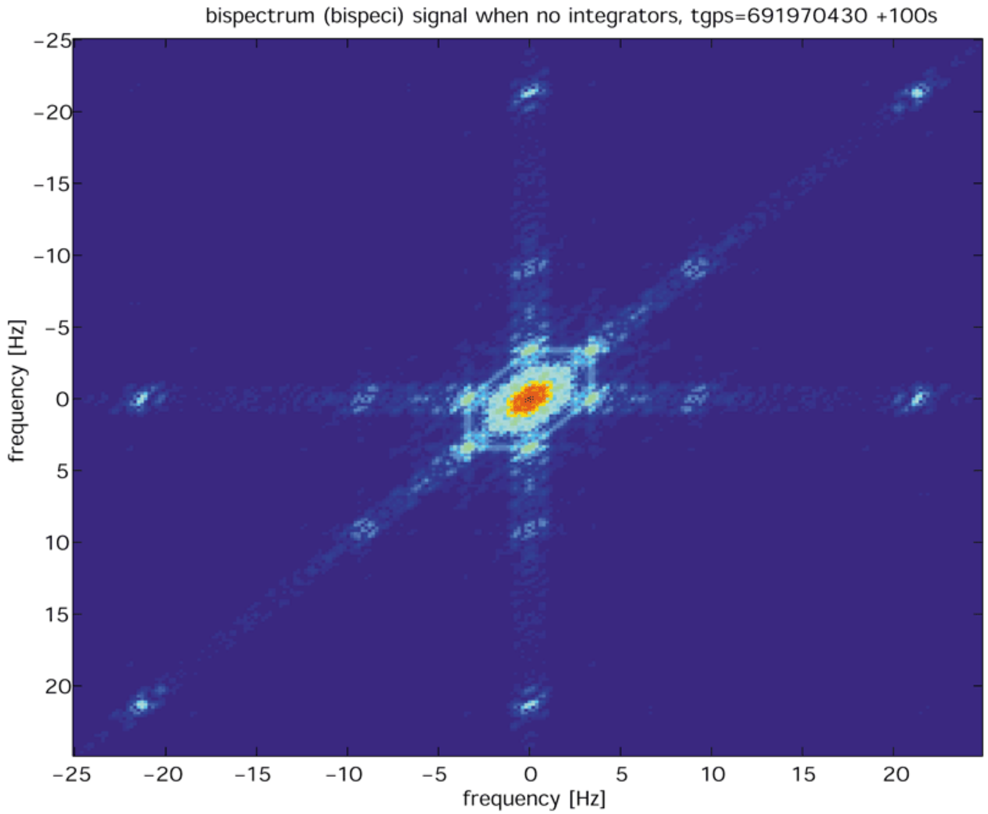
Represents the signal energy in the time-frequency domain centred in (t,ν).

•To analize the system linearity bispectrum and bicoherence need to be taken into account:

•If $S_{3x}=0$ the process is Gaussian and linear

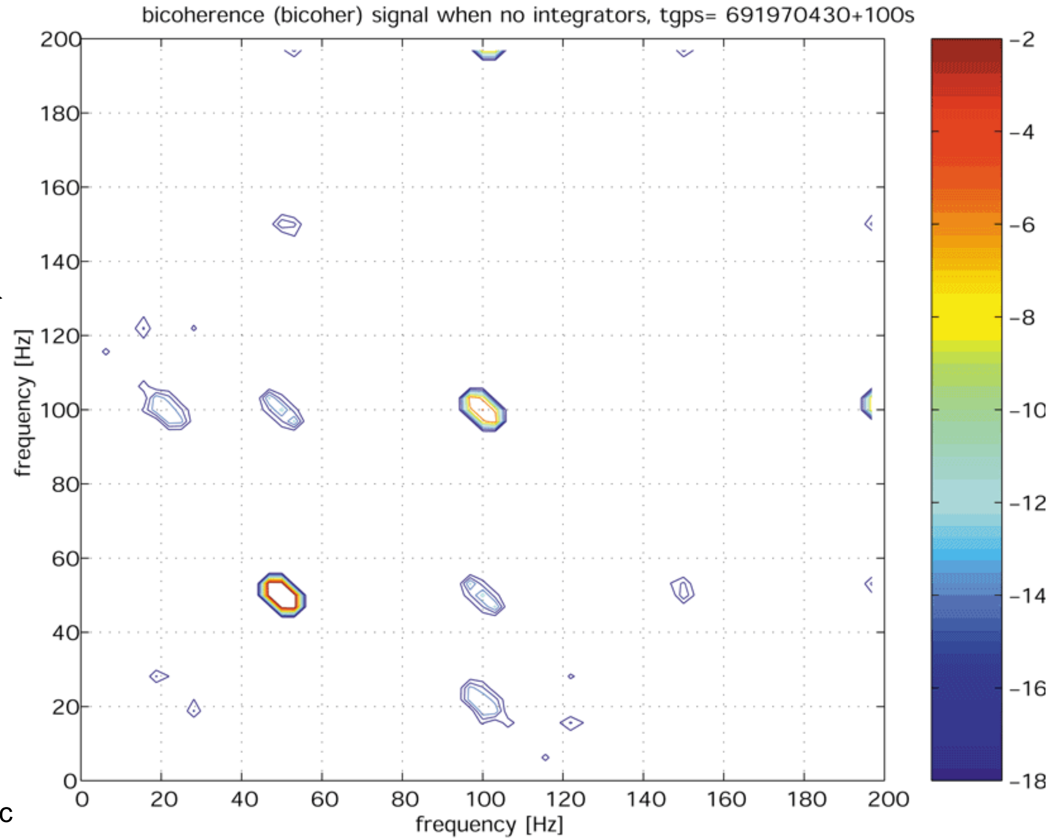
•If $S_{3x}\neq 0$ the process is not Gaussian and

- •if c_{3x} is constant - the process is linear
- •if c_{3x} is not constant - the process is not linear



Bispectrum

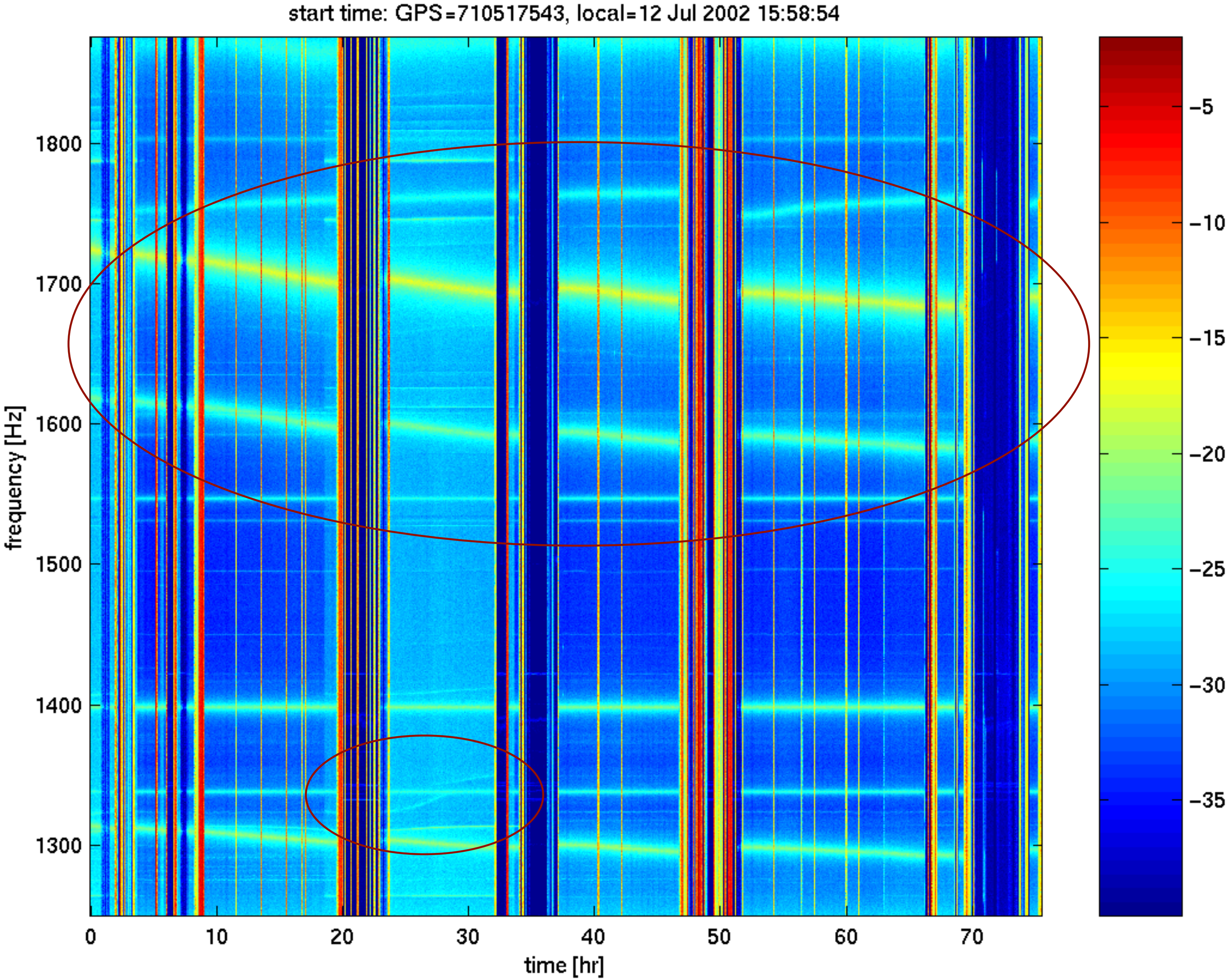
Low frequency noise coupled at higher frequencyes



Bicoherence

A nonlinearity of the 50 Hz with its armonics is observed.
There is present a big coupling between the 20 Hz and the 100 Hz and a smaller one between the 20 and 30 Hz.

Spectrogram:



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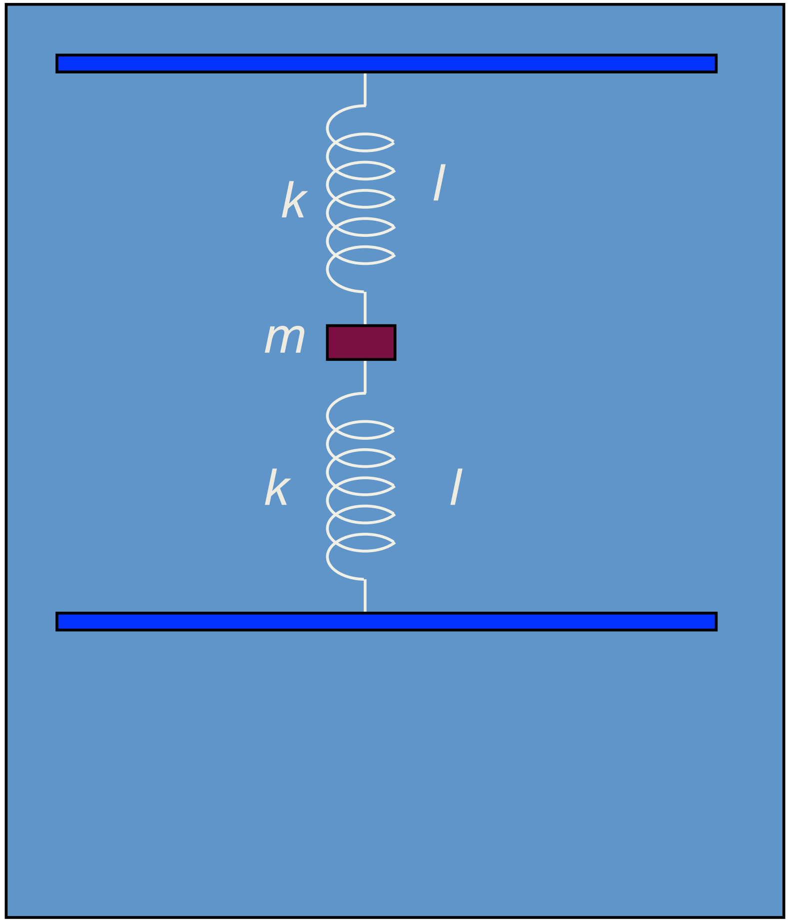
Noise in Physical Sys

Let's look at an example of non-linear oscillator:

the Duffing Oscillator

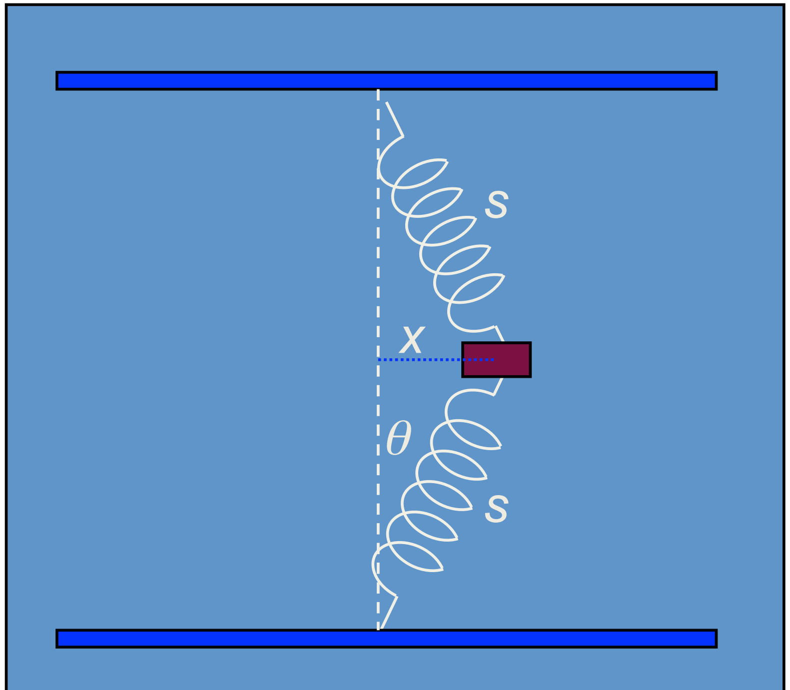
$$\ddot{x} + \delta\dot{x} + \beta x + \alpha x^3 = \gamma\cos\omega t$$

A two springs system



- A mass is held between two springs.
  - Spring constant *k*
  - Natural length *l*
- Springs are on a horizontal surface.
  - Frictionless
  - No gravity

Transverse Displacement



$$s = \sqrt{l^2 + x^2}$$

- The force for a displacement is due to both springs.
  - Only transverse component
  - Looks like its harmonic

$$F = -2k\left(\sqrt{l^2 + x^2} - l\right)\sin\theta$$

$$= -2k\left(\sqrt{l^2 + x^2} - l\right)\frac{x}{\sqrt{l^2 + x^2}}$$

$$= -2kx\left(1 - \frac{1}{\sqrt{1 + x^2/l^2}}\right)$$

Purely Nonlinear

- The force can be expanded as a power series near equilibrium.
  - Expand in *x/l*

$$F = -2kl\frac{x}{l}\left(1 - \frac{1}{\sqrt{1 + x^2/l^2}}\right)$$

- The lowest order term is non-linear.

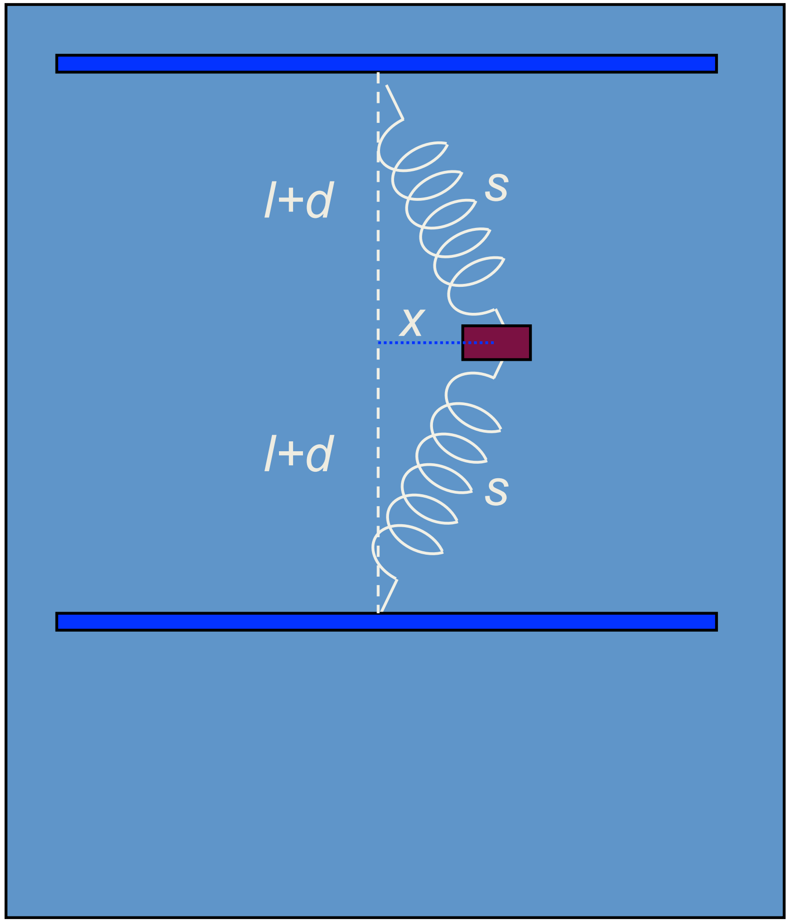
$$F \cong -kl\left(\frac{x}{l}\right)^3 + \ldots$$

$$\Downarrow$$

- Quartic potential
  - Not just a perturbation

$$V \cong \frac{k}{4l^2}x^4 + \ldots$$

Mixed Potential



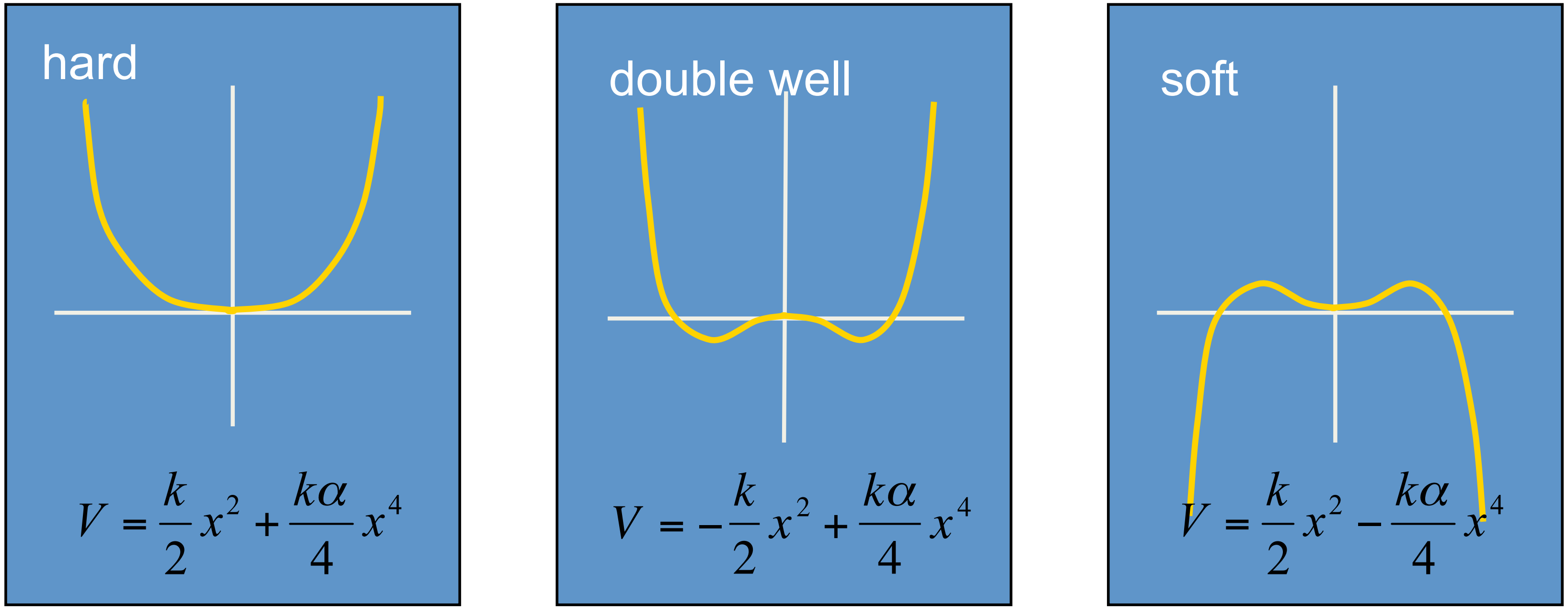
- Typical springs are not at natural length.
  - Approximation includes a linear term

$$F \cong -\frac{2kd}{l}x - \frac{k(l-d)}{l^3}x^3 + \dots$$

$$V \cong \frac{kd}{l}x^2 + \frac{k(l-d)}{4l^3}x^4 + \dots$$

Quartic Potentials

- The sign of the forces influence the shape of the potential.



Driven System

- Assume a more complete, realistic system.
  - Damping term
  - Driving force

$$m\ddot{x} = -\beta\dot{x} - kx - k\alpha x^3 + f\cos\omega t$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x + \alpha\omega_0^2 x^3 = f\cos\omega t$$

- Rescale the problem:
  - Set *t* such that $\omega_0^2 = k/m = 1$
  - Set *x* such that $k\alpha/m = 1$

- This is the Duffing equation

$$\ddot{x} + \gamma\dot{x} + x \pm x^3 = f\cos\omega t$$

Steady State Solution

- Try a solution, match terms

$$x(t) = A(\omega)\cos[\omega t - \theta(\omega)]$$

$$\ddot{x} + \gamma\dot{x} + x \pm x^3 = f\cos\omega t$$

$$A(1-\omega^2)\cos(\omega t - \theta) - A\gamma\omega\sin(\omega t - \theta) \pm A^3\cos^3(\omega t - \theta) = f\cos\omega t$$

trigonometric identities

$$\cos^3(\omega t - \theta) = \tfrac{3}{4}\cos(\omega t - \theta) + \tfrac{1}{4}\cos 3(\omega t - \theta)$$

$$f\cos\omega t = f\cos\theta\cos(\omega t - \theta) - f\sin\theta\sin(\omega t - \theta)$$

$$\begin{aligned}&[A(1-\omega^2 \pm \tfrac{3}{4}A^2) - f\cos\omega t]\cos(\omega t - \theta)\\ &+[-A\gamma\omega + f\sin\omega t]\sin(\omega t - \theta)\\ &\pm \tfrac{1}{4}A^3\cos 3(\omega t - \theta)\\ &= 0\end{aligned}$$

$$f\cos\omega t = A(1-\omega^2 \pm \tfrac{3}{4}A^2)$$

$$f\sin\omega t = A\gamma\omega$$

$$\pm \tfrac{1}{4}A^3\cos 3(\omega t - \theta) \approx 0$$

Amplitude Dependence

- Find the amplitude-frequency relationship.
  - Reduces to forced harmonic oscillator for A → 0

$$f^2 = A^2[(1-\omega^2)^2 + (\gamma\omega)^2]$$

$$f^2 \cos^2 \omega t = A^2(1-\omega^2 \pm \tfrac{3}{4}A^2)^2$$

$$f^2 \sin^2 \omega t = A^2\gamma^2\omega^2$$

$$f^2 = A^2[(1-\omega^2 \pm \tfrac{3}{4}A^2)^2 + \gamma^2\omega^2]$$

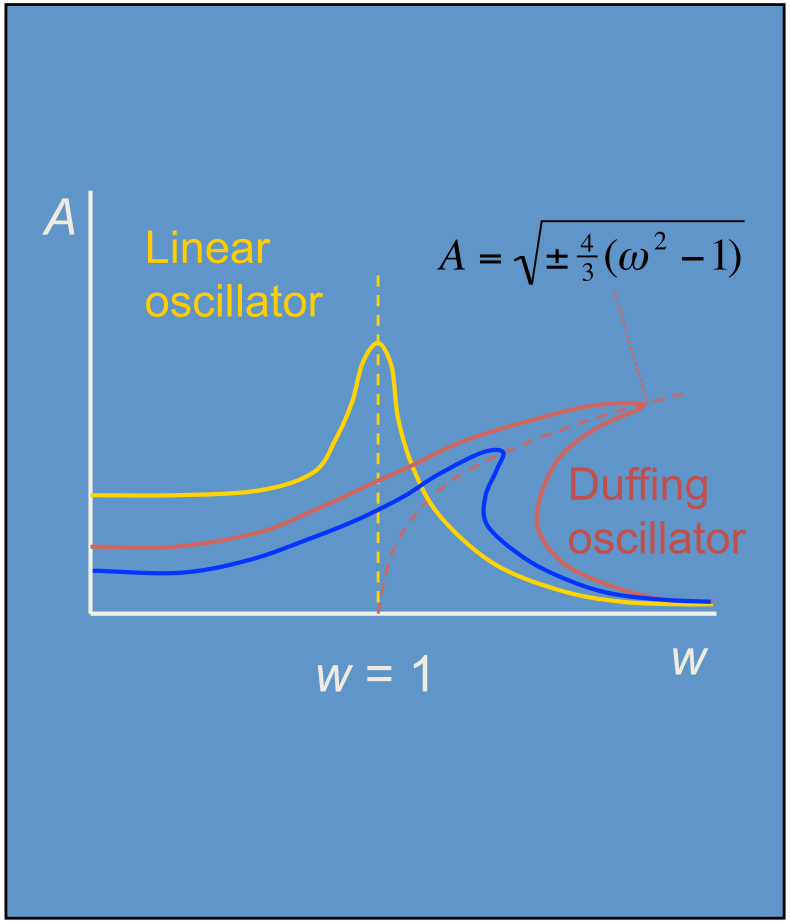
- Find the case for minimal damping and driving force.
  - f, γ both near zero
  - Defines resonance condition

$$0 = A^2[(1-\omega^2 \pm \tfrac{3}{4}A^2)^2 + 0]$$

$$0 = 1-\omega^2 \pm \tfrac{3}{4}A^2$$

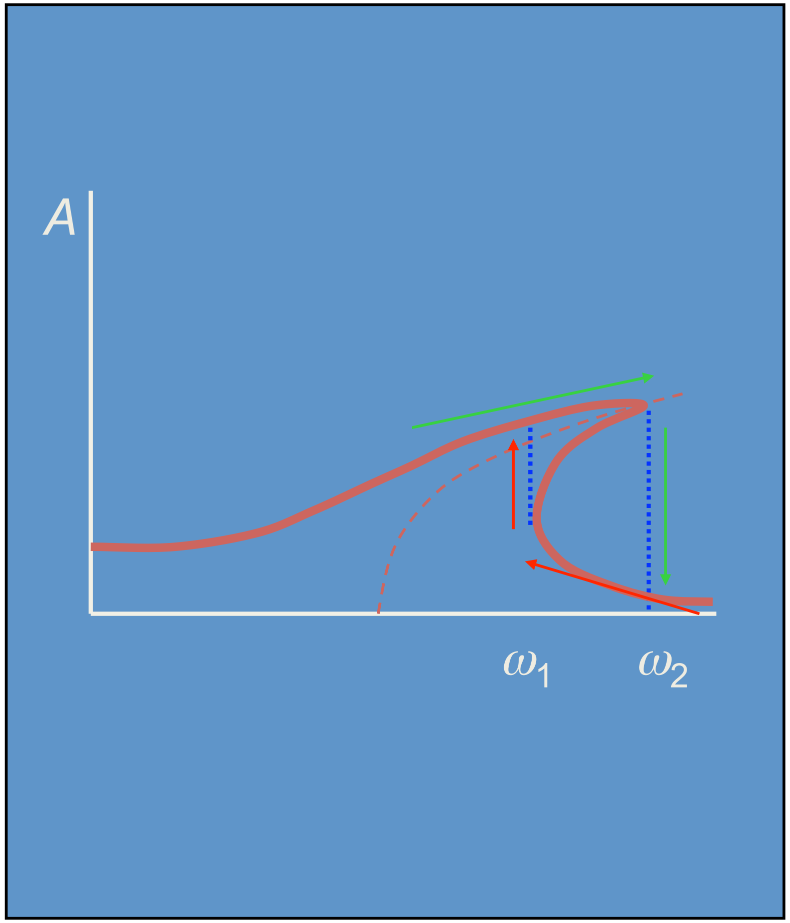
$$A(\omega) = \sqrt{\pm \tfrac{4}{3}(\omega^2 - 1)}$$

Nonlinear Resonance Frequency



- The resonance frequency of a linear oscillator is independent of amplitude.
- The resonance frequency of a Duffing oscillator increases with amplitude.

… brings to hysteresis



- A Duffing oscillator behaves differently for increasing and decreasing frequencies.
  - Increasing frequency has a jump in amplitude at ω_2
  - Decreasing frequency has a jump in amplitude at ω_1
- This is hysteresis.

Nonlinear Resonance

(in general…)

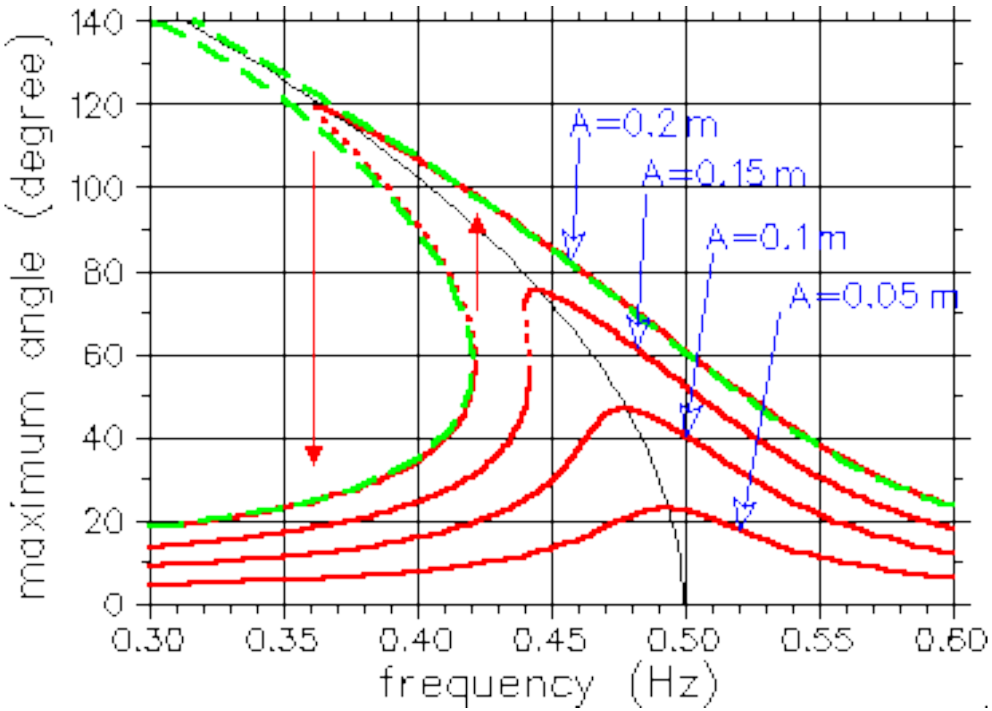
Nonlinear resonance seems not to be so much different from the (linear) resonance of a harmonic oscillator. But both, the dependency of the eigenfrequency of a nonlinear oscillator on the amplitude and the nonharmoniticity of the oscillation lead to a behavior that is impossible in harmonic oscillators, namely, the foldover effect and superharmonic resonance, respectively.

Both effects are especially important in the case of weak damping.

The foldover effect

The foldover effect got its name from the bending of the resonance peak in a amplitude versus frequency plot. This bending is due to the frequency-amplitude relation which is typical for nonlinear oscillators.

$$g = 9.81 m/\sec^2, l = 1m, \gamma = 0.4 \sec^{-1}$$



The pendulum eq.:

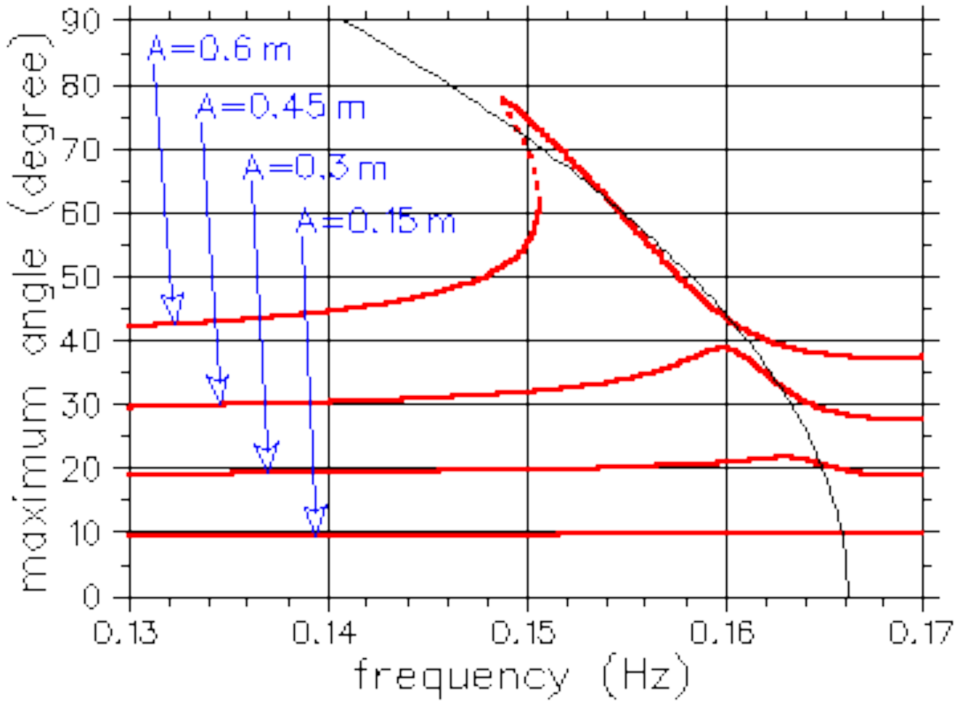
$$\ddot{\varphi} = -\gamma\dot{\varphi} - \omega_0^2 \sin\varphi + f\cos\omega t$$

$$\omega_0^2 = \frac{g}{l}$$

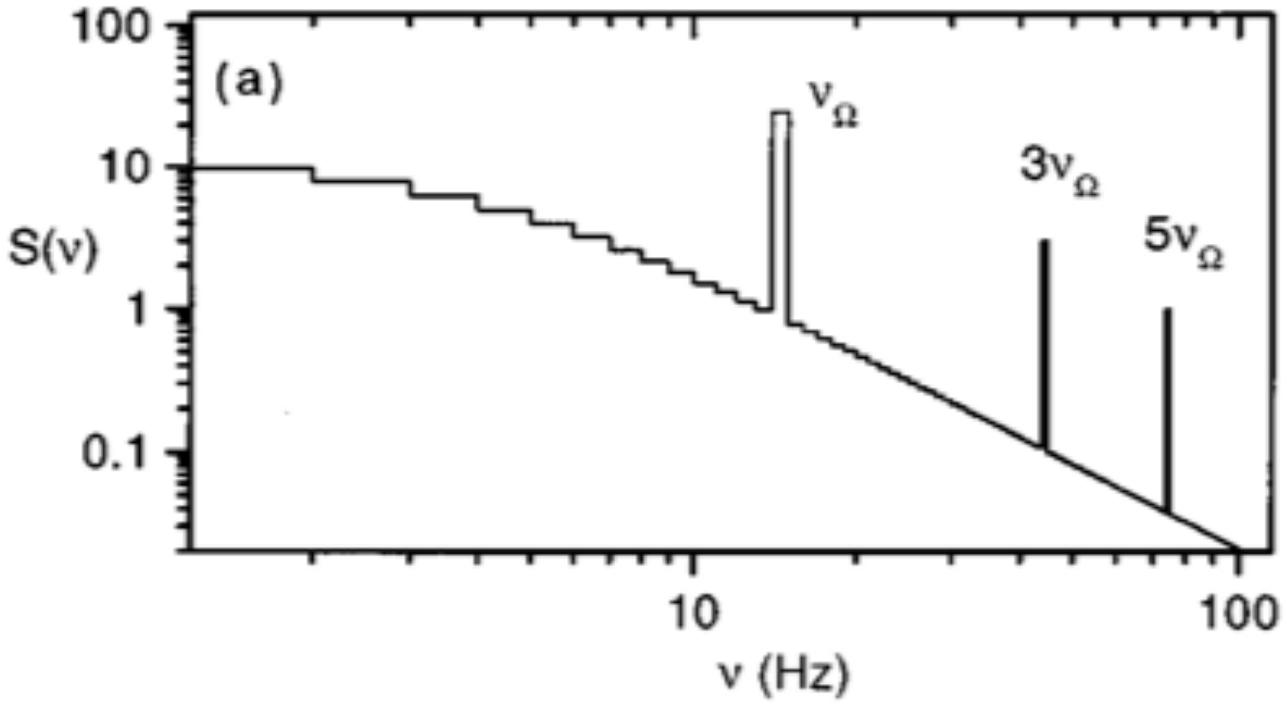
The superharmonic resonance

Nonlinear oscillators do not oscillate sinusoidal.
Superharmonic resonance is simply the resonance with one of this higher harmonics of a nonlinear oscillation. In an amplitude/frequency plot appear additional resonance peaks. In general, they appear at driving frequencies which are integer fractions of the fundamental frequency.

$$g = 9.81m/\sec^2, l = 1m, \gamma = 0.1\sec^{-1}$$



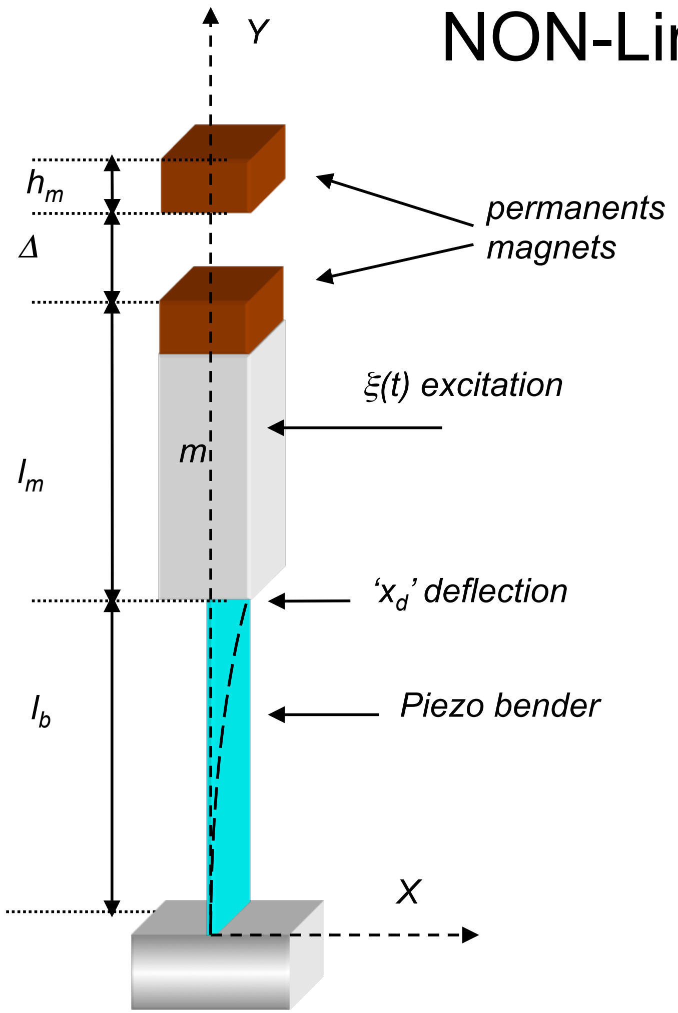
Overdamped Duffing



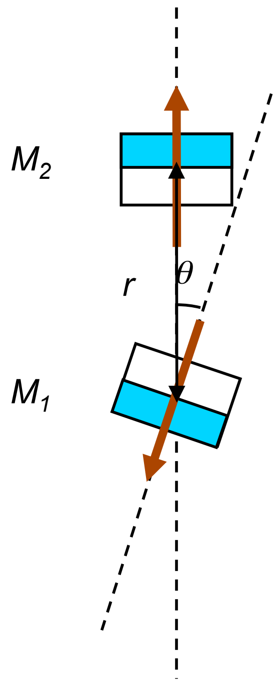
Gammaitoni et al. Reviews of Modern Physics 1998

Noise energy harvesting

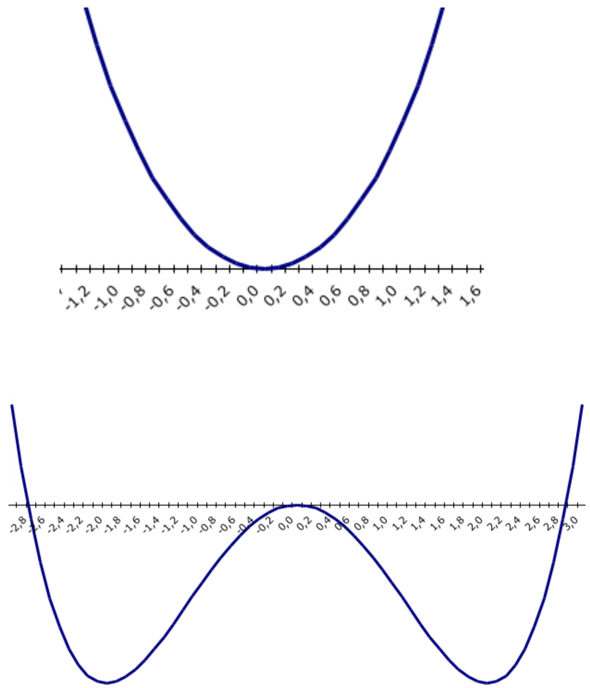
NON-Linear mechanical oscillators



NON-Linear Inverted pendulum



b)



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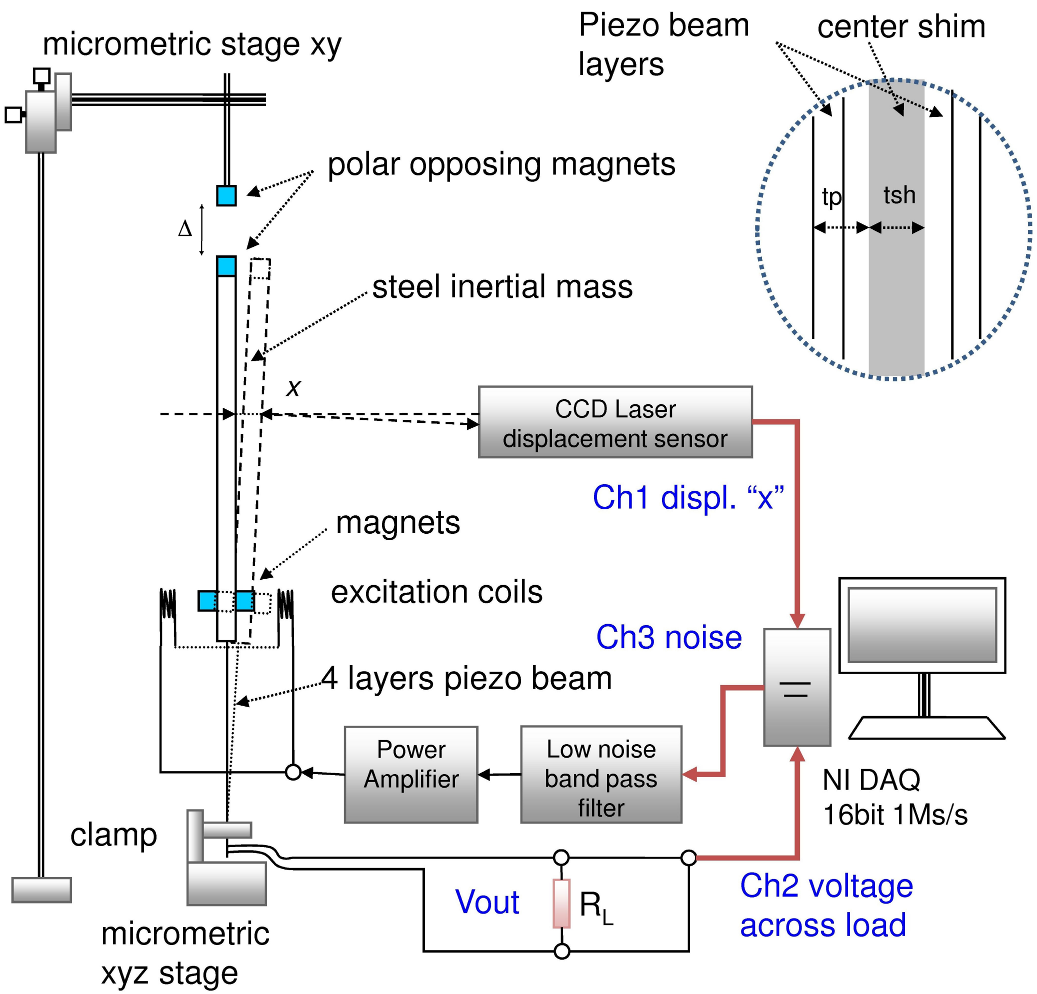

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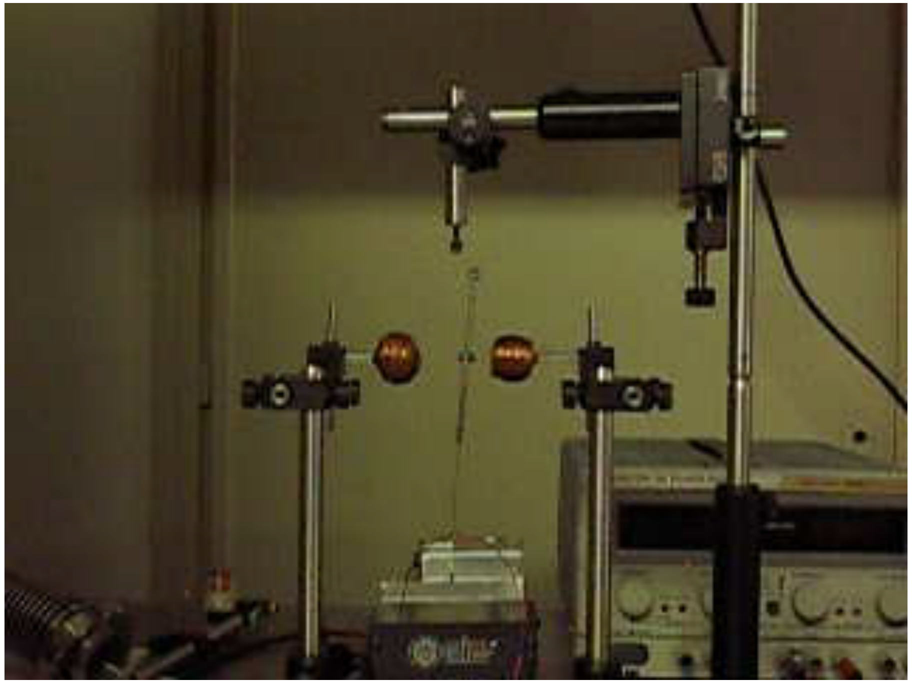
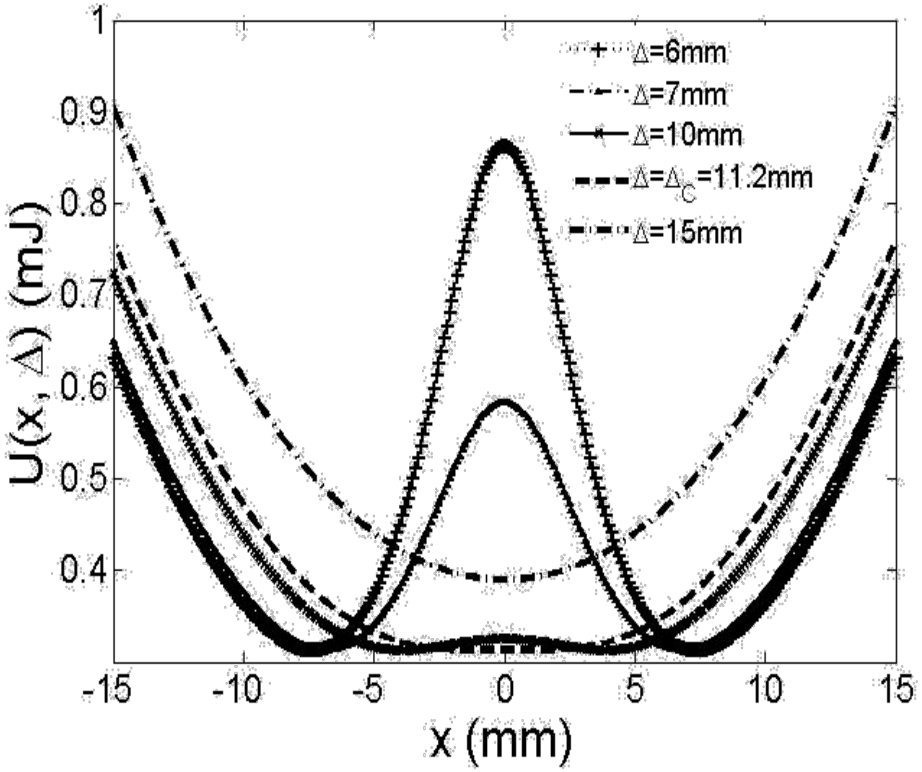


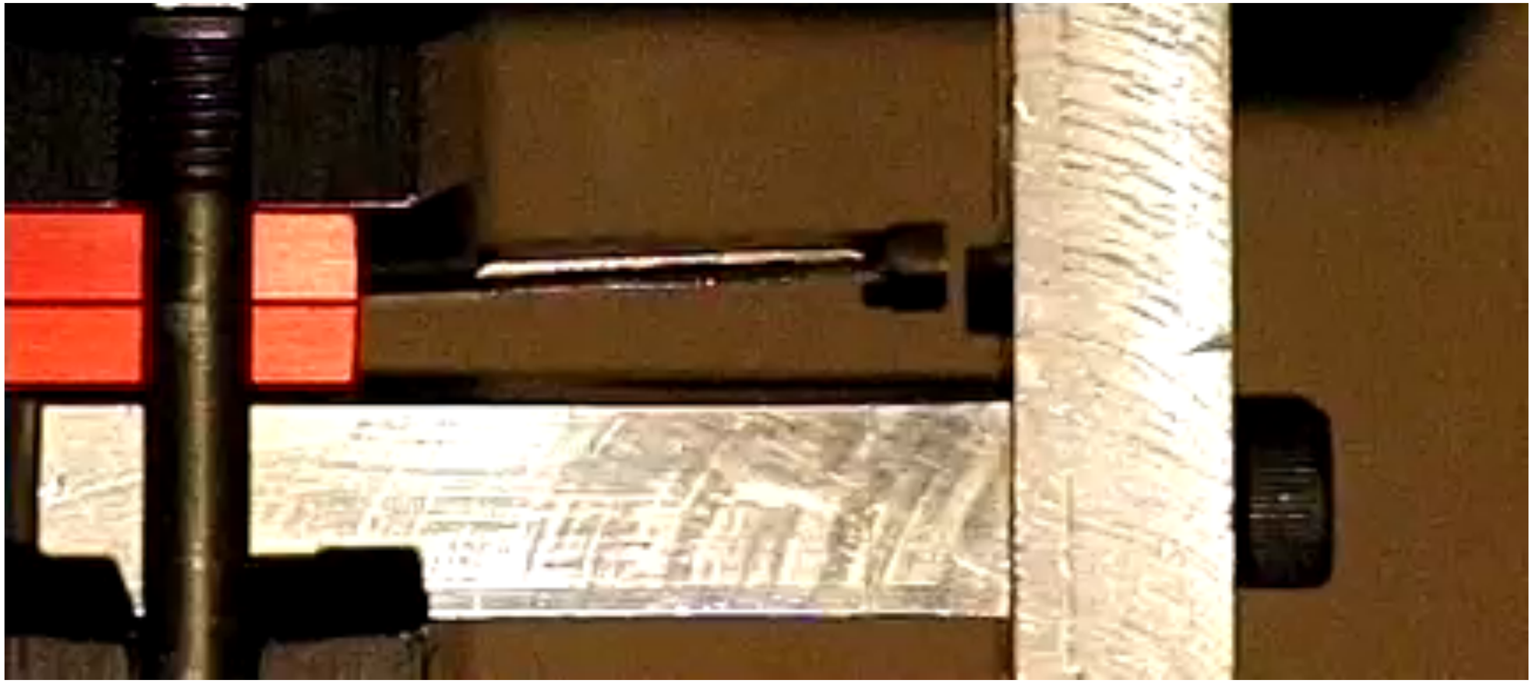


http://www.nipslab.org/node/1676

Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni
Physical Review Letters, 102, 080601 (2009)

Noise energy harvesting

NON-Linear mechanical oscillators



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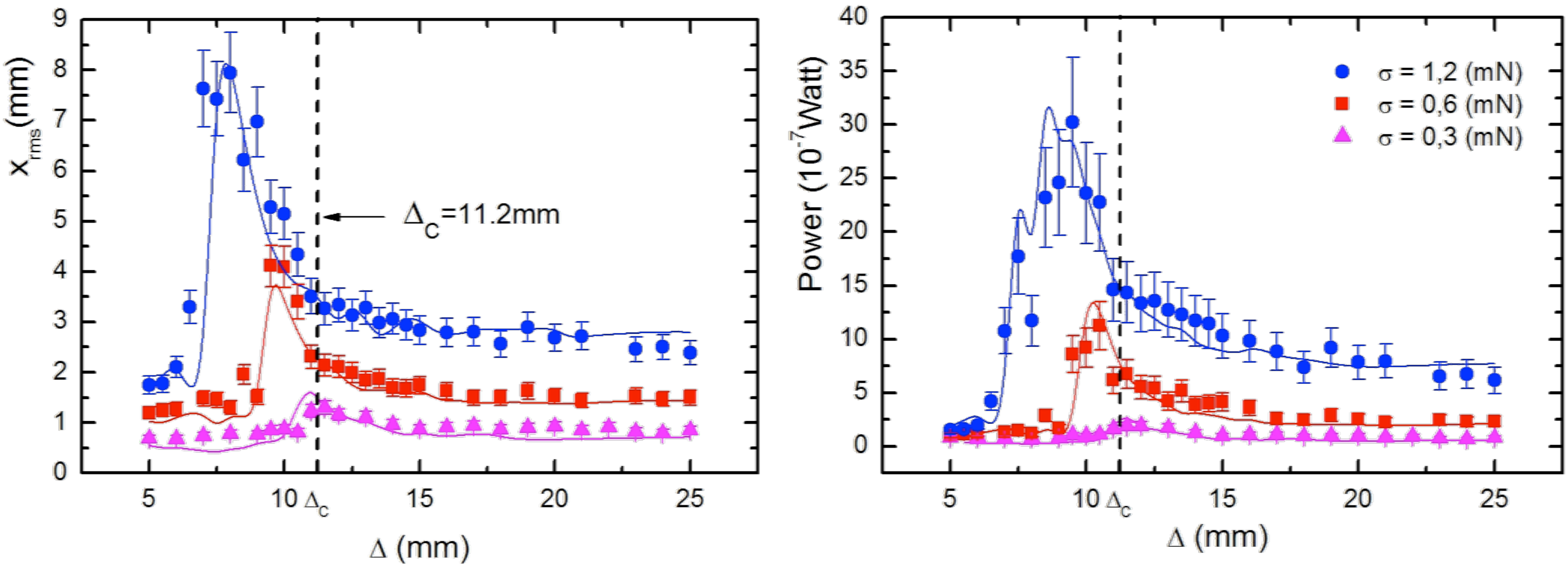
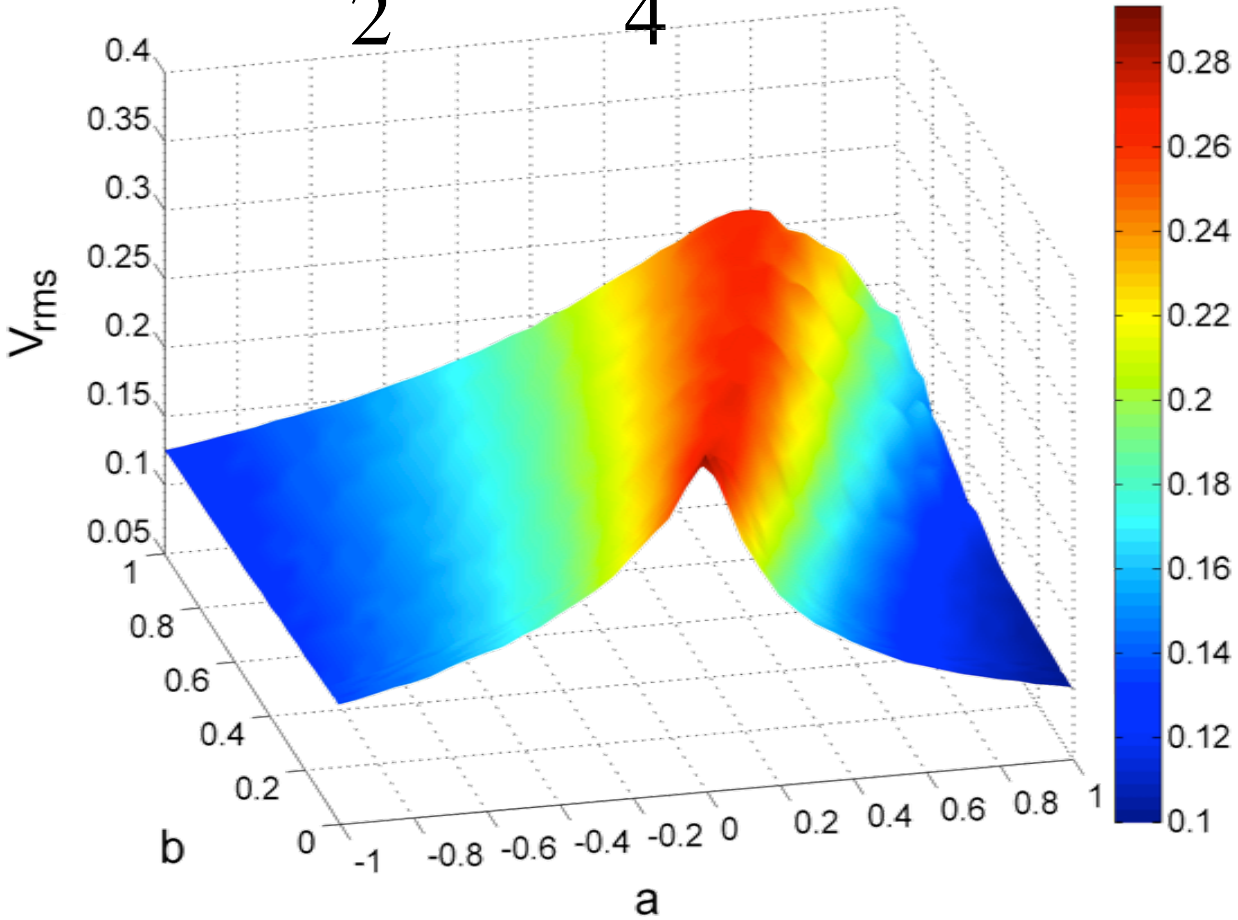


Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni , Physical Review Letters, 102, 080601 (2009)

Noise energy harvesting

Non-linear systems

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$ Duffing potential



L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

Noise energy harvesting

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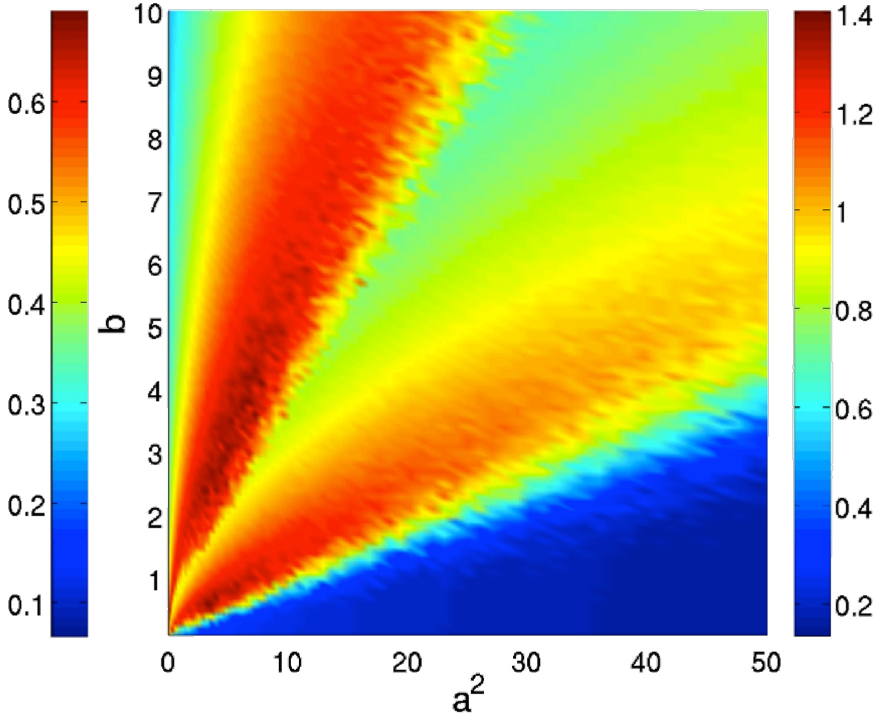
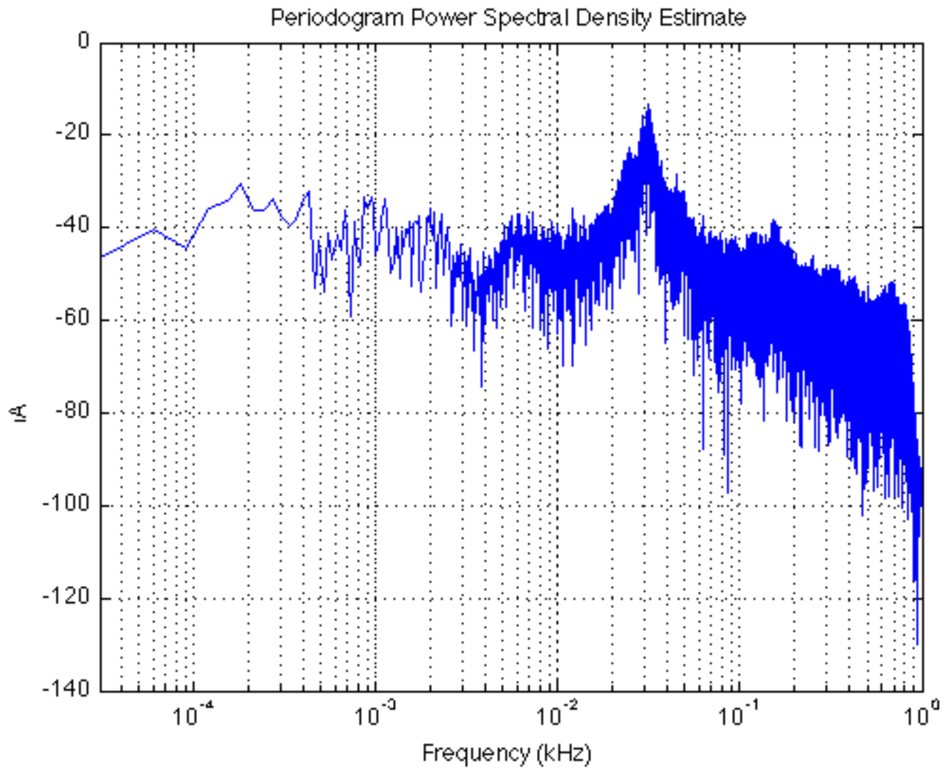
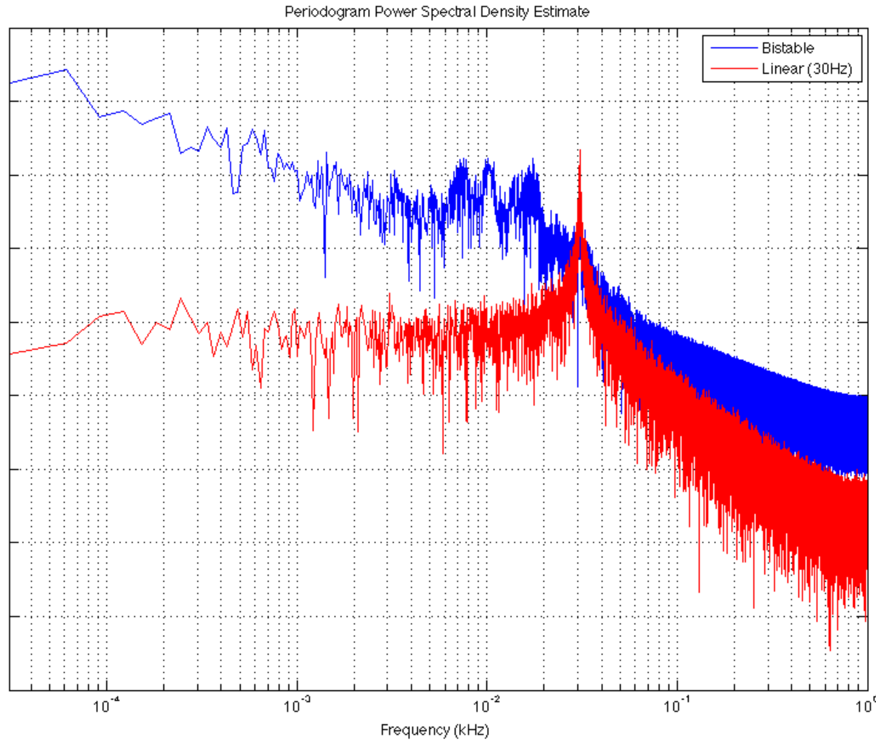


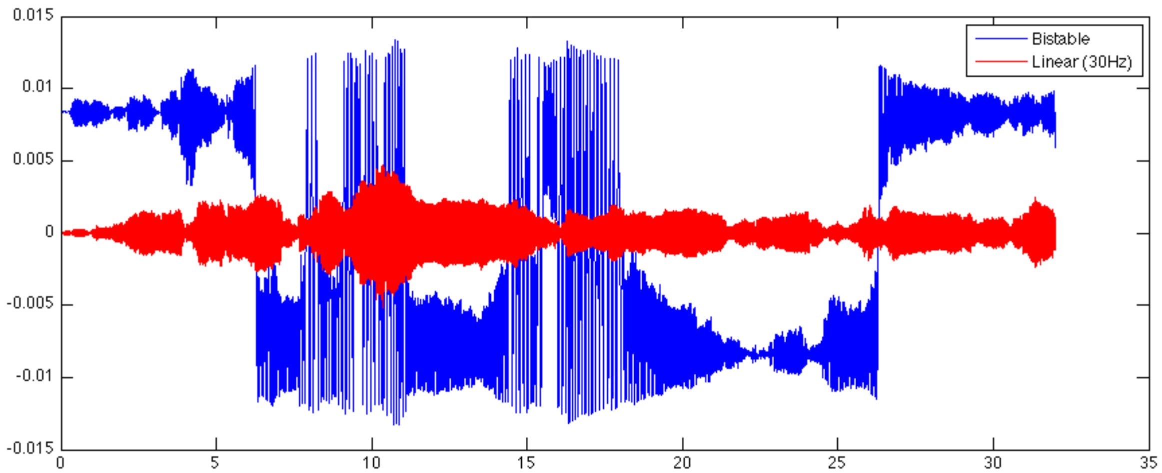
$$b_{MAX} = \frac{a^2}{4D\ \log(\tau_p)}$$

L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

A compared response







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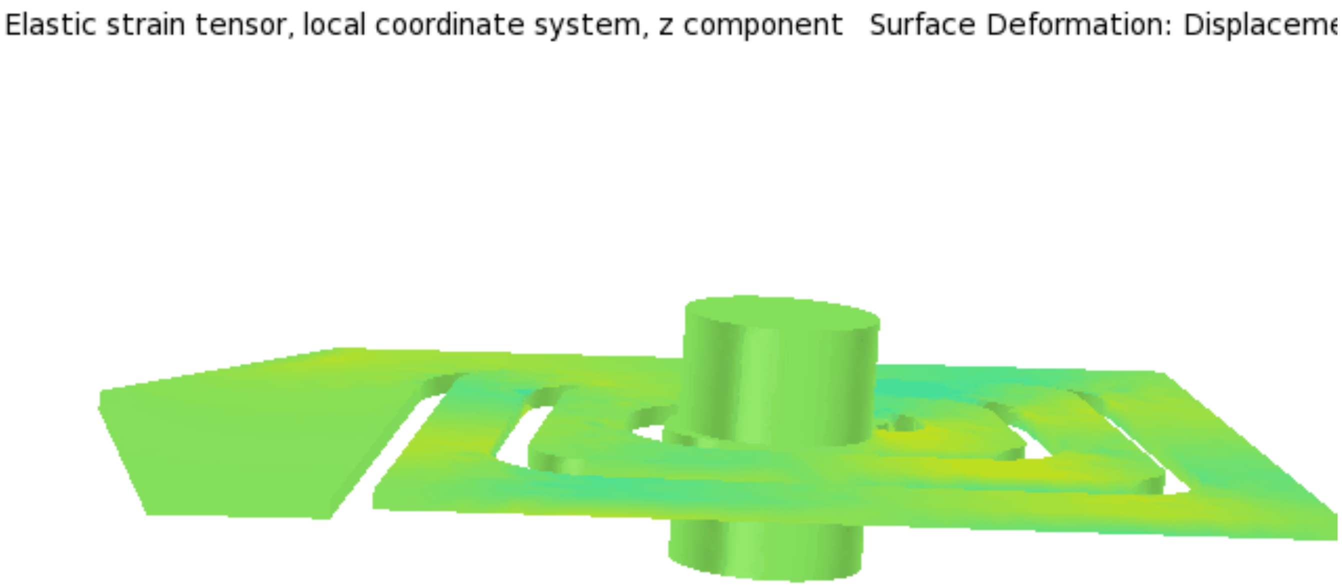
Non-linear systems next evolutions…



Noise energy harvesting

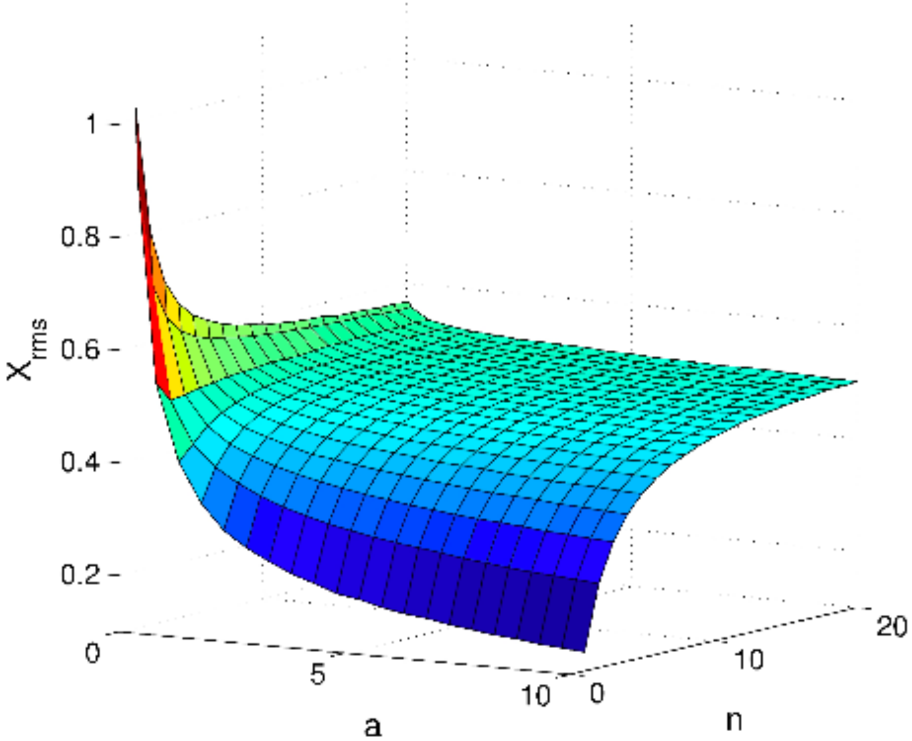
Only bistability???

A more general monostable potential…

$$U(x) = ax^{2n}$$

$$with : a > 0$$

$$n = 1,2,...$$



$$a_{th} \approx \frac{D}{4} = \sigma^2 \tau$$

L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

In conclusion: to think about…

1) Non resonant (i.e. non-linear) mechanical oscillators can outperform resonant (i.e. linear) ones*

2) Non-linear systems are more difficult to treat

3) Bistability is not the only nonlinearity available… see:
   L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

* **wisepower technology** patent. For more info see: www.nipslab.org, www.wisepower.it

Further readings

Energy harvesting for mobile systems:

Paradiso, J., A., Starner, T., *Energy Scavenging for Mobile and Wireless Electronics*, IEEE Pervasive Computing, Vol. 4, No. 1, February 2005, pp. 18-27.

Roundy, S., Wright, P.K., Rabaey, J.M. Energy Scavenging for Wireless Sensor Networks, 2003 Kluwer Academic Publishers, Boston MA.

Vibration Energy harvesting:

Meninger, S. et al. Vibration-to-Electric Energy Conversion, IEEE Trans. Very Large Scale Integration (VLSI) Systems, vol. 9, no. 1, 2001, pp. 64–76.

Nonlinear Vibration Energy harvesting:

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F. Cottone, H. Vocca, and L. Gammaitoni, Phys. Rev. Lett. 102, 080601, (2009)

Noise and fluctuations:

H.L. Pecseli, *Fluctuations in Physical Systems*, Cambridge Univerity Press, 2000

Nonlinear dynamics:

S.H. Strogatz, *Nonlinear Dynamics and Chaos,* Addison-Wesley, 1994