NANO-CTM

# Current from hot spots

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**University of Geneva**

NiPS Summer School 2011
"Energy Harvesting at micro and nanoscale"
Perugia, Aug. 1-6 (2011)

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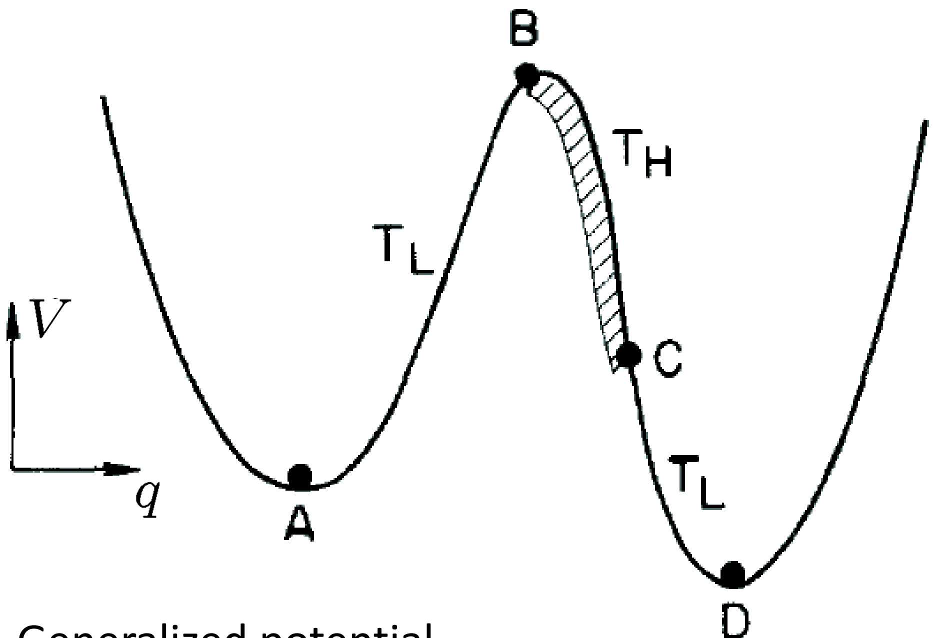
Andrew Jordan

NANOPOWER

# Relative stability controlled by hot spots

R. Landauer, J. Stat. Phys. 53, 233 (1988).




Probability current

$$j = v(q)\rho(q) - D(q)\frac{d\rho(q)}{dq}$$

$$v(q) = -\mu\frac{dV(q)}{dq}$$

Specifically

$$D = \mu kT_L, \quad q\,in\,L$$
$$D = \mu kT_H, \quad q\,in\,H$$

Stationary state

$$j = 0 \quad \Longrightarrow$$

$$\rho(q) = \rho_0\, e^{-\Psi(q)}$$

Generalized potential

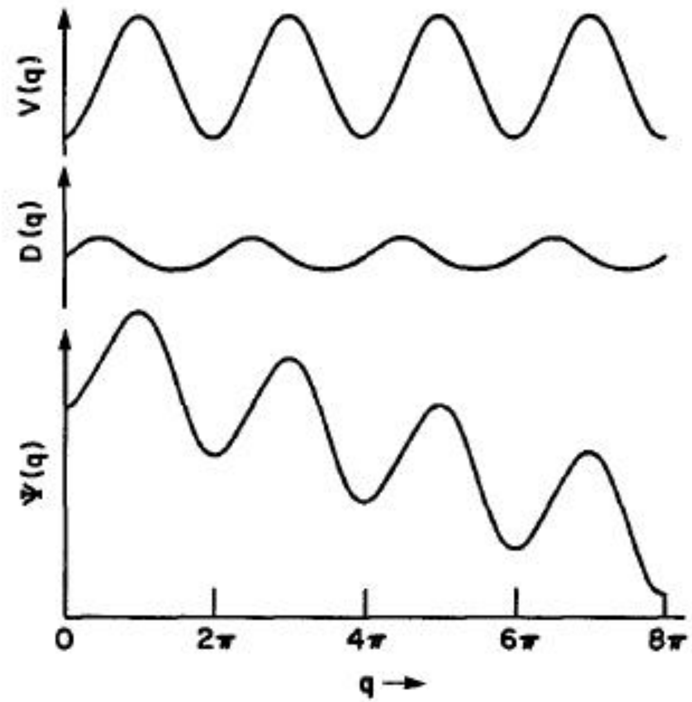
$$\Psi(q) = -\int_0^q dp\,\frac{v(p)}{D(p)} \quad \Longrightarrow$$

$$\frac{\rho_A}{\rho_B} = e^{-\frac{V_A - V_D}{kT_L}}\, e^{-(V_B - V_C)\left(\frac{1}{kT_H} - \frac{1}{kT_L}\right)}$$

Hot spots in rarely visited parts of the phase space influence relative stability

# Noise induced currents

M. B., Z. Phys. B 68, 161 (1987); [N.G. van Kampen, IBM J. Res. Dev. 32, 107 (1988)]




Periodic potential

$$V(q) = V(q + 2\pi)$$

Periodic statedependent diffusion constant

$$D(q) = D(q + 2\pi)$$

Specifically

$$V(q) = V_0(1 - \cos(q))$$
$$D^{-1}(q) = D_0^{-1}(1 - \alpha\cos(q - \phi))$$
$$D_0 = \mu kT$$

Probability current

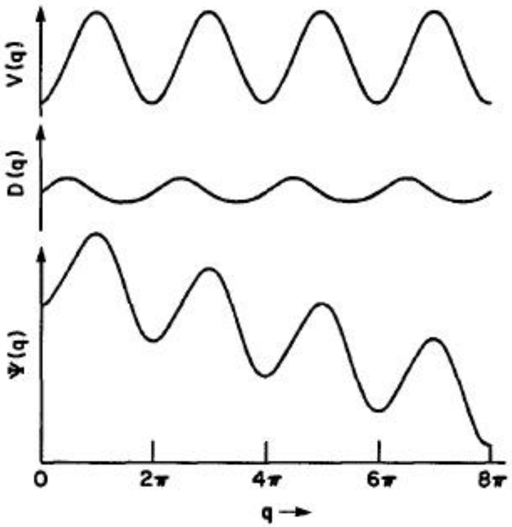
$$j = v(q)\rho(q) - D(q)\,d\rho(q)/dq\,, \quad v(q) = -\mu dV(q)/dq$$

Generalized potential

$$\Psi(q) = -\int^q dp\,\frac{v(p)}{D(p)}$$

# Noise induced currents

M. B., Z. Phys. B 68, 161 (1987)




$$\Psi(q) = -\int^{q} dp \, \frac{v(p)}{D(p)}$$

$$V(q) = V_0(1 - \cos(q))$$

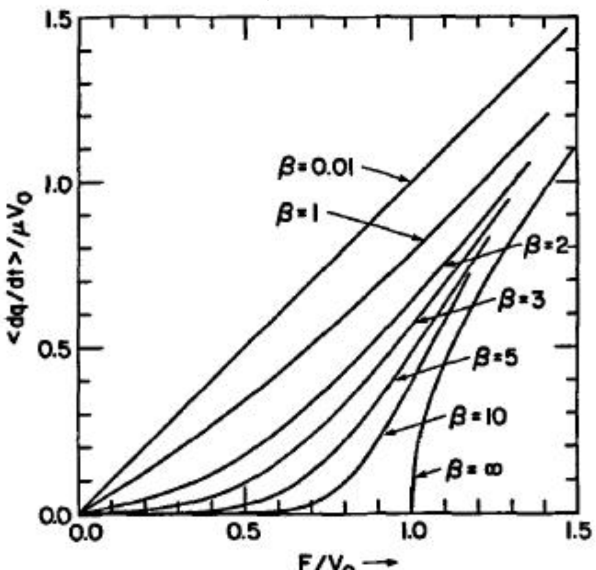
$$D^{-1}(q) = D_0^{-1}\,(1 - \alpha\cos(q - \phi))$$

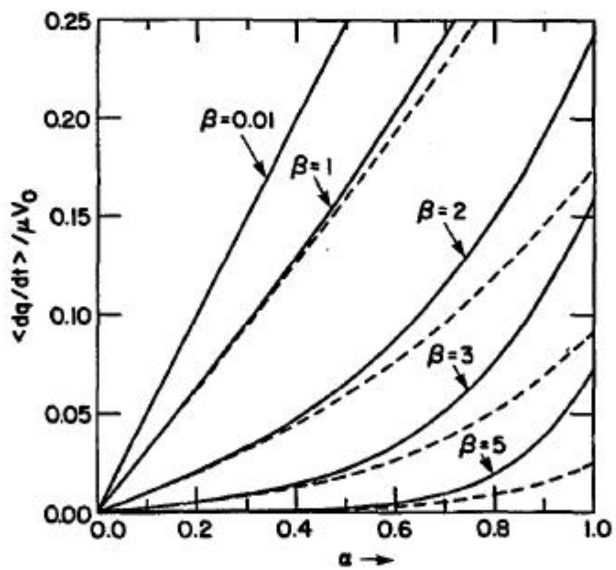
$$\Longrightarrow$$

$$\Psi(q) = \Psi(q + 2\pi) + 2\pi\,\Delta$$

$$\Delta = \frac{\mu V_0}{D_0}\frac{\alpha}{2}\sin(\phi) \qquad D_0 = \mu\, kT$$

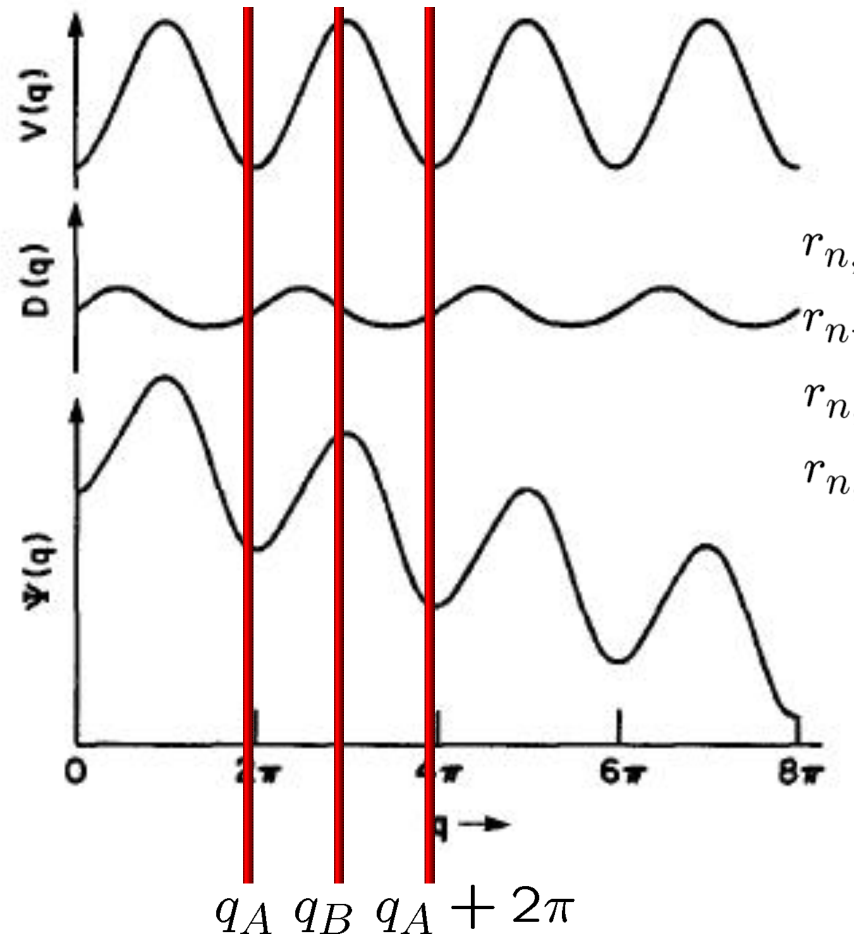
$$\beta = V_0/kT$$



# Noise induced currents

M. B., Z. Phys. B 68, 161 (1987)




Activation rates

$$r_{n,n+1} = \omega \exp(-(\Psi(q_B) - \Psi(q_A))$$

$$r_{n+1,n} = \omega \exp(-(\Psi(q_B) - \Psi(q_A + 2\pi))$$

$$r_{n,n+1} / r_{n+1,n} = \exp(\Psi(q_A + 2\pi) - (\Psi(q_A))$$

$$r_{n,n+1} / r_{n+1,n} = \exp(-2\pi\Delta)$$

$$\Delta = \frac{\mu V_0}{D_0} \frac{\alpha}{2} \sin(\phi)$$

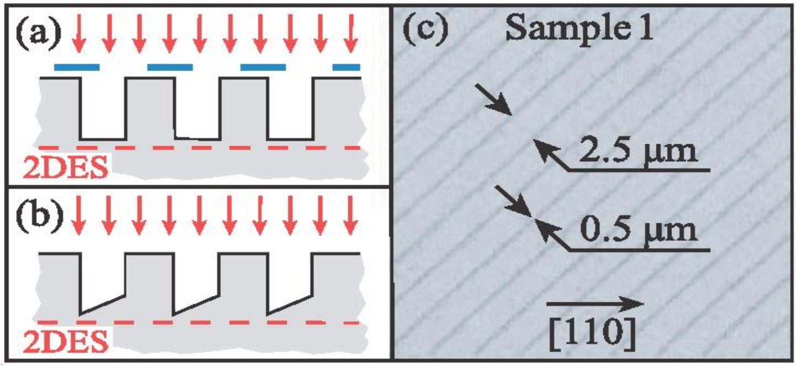
Underdamped case

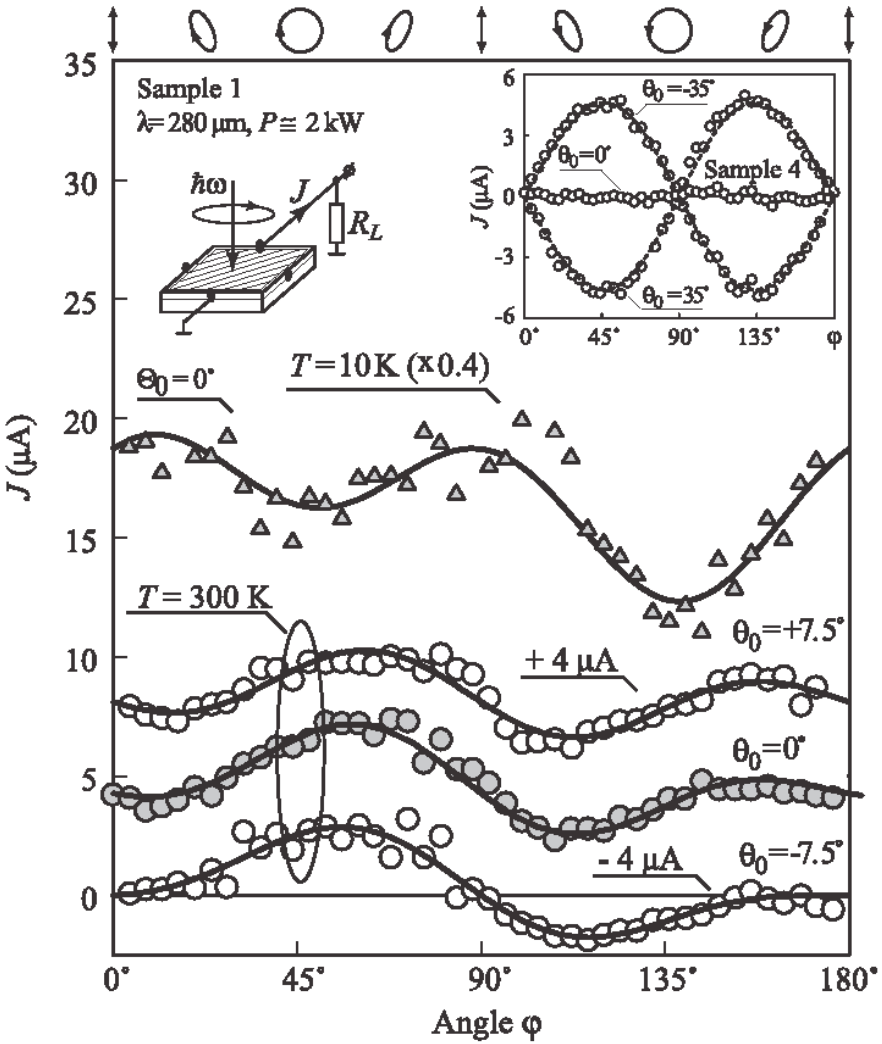
Y. M. Blanter and M. B., Z. PRL 81, 4040 (1998)

# Experiments on superlattices

P. Olbrich, et al., Phys. Rev. Lett. 103, 090603 (2009)

P. Olbrich, et al., Phys. Rev. B 83, 165320 (2011)




THz, $\lambda = 280\,\mu m$

$\phi$ helicity

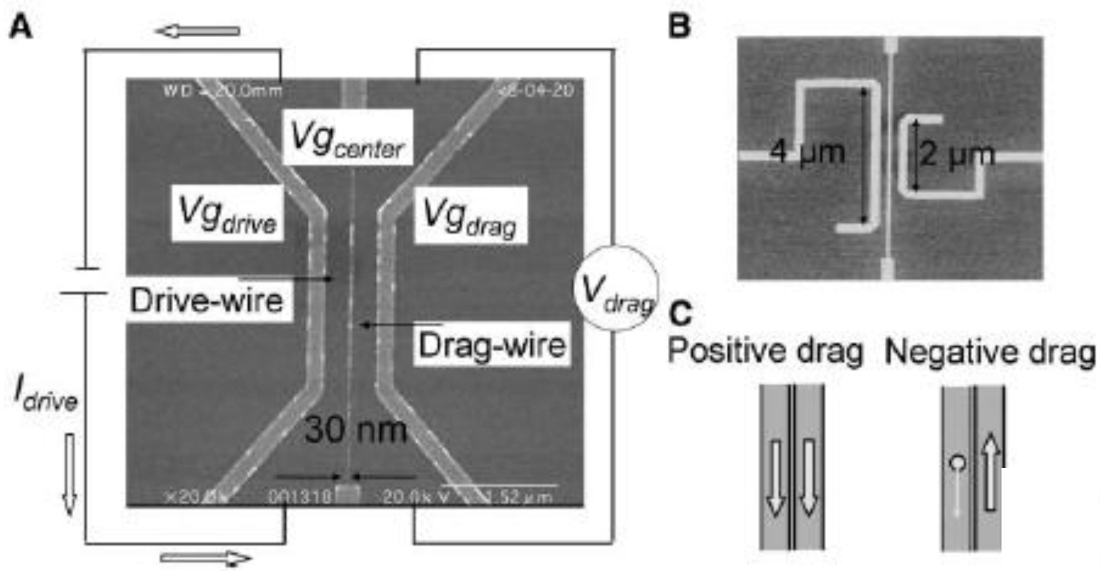
$\Theta$ angle of incidence

# Noise induced transport via Coulomb interaction

## Coulomb drag

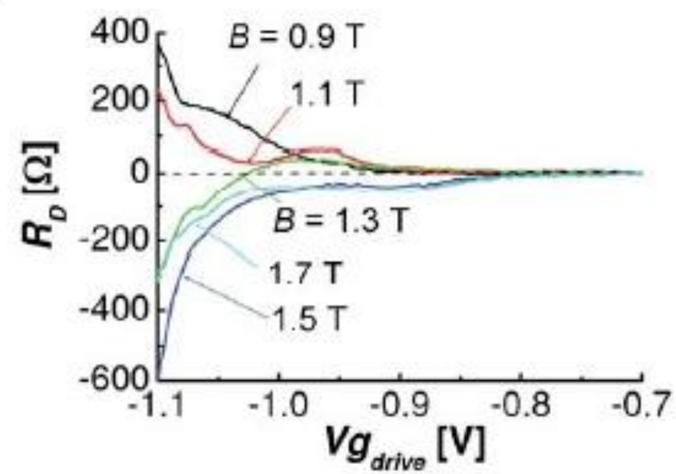
Example form the mesoscopic literature

M. Yamamoto et al. (Tarucha) , Science 313, 204  (2006).



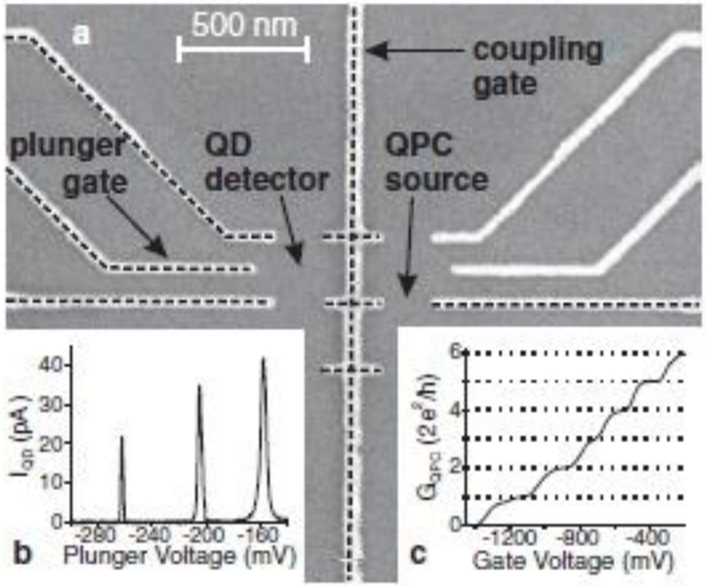

Drag resistance

$$R_D = V_{drag}/I_{drive}$$

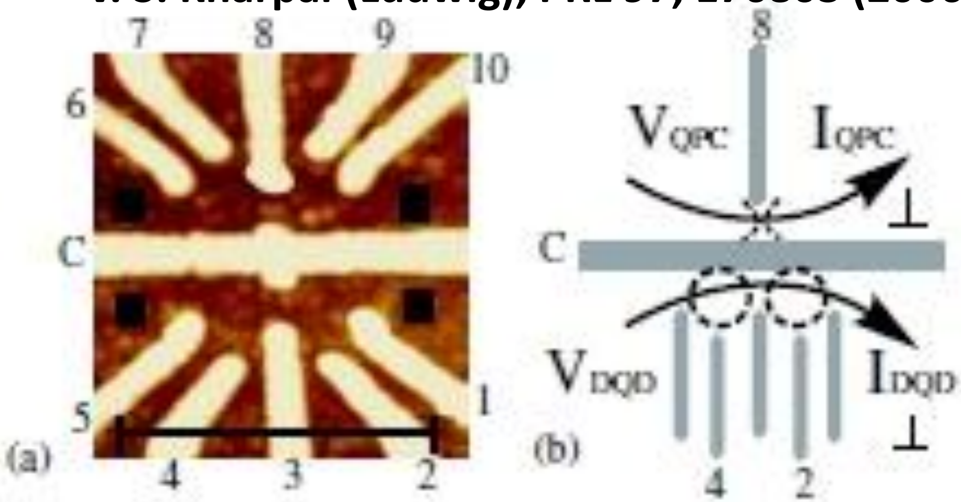


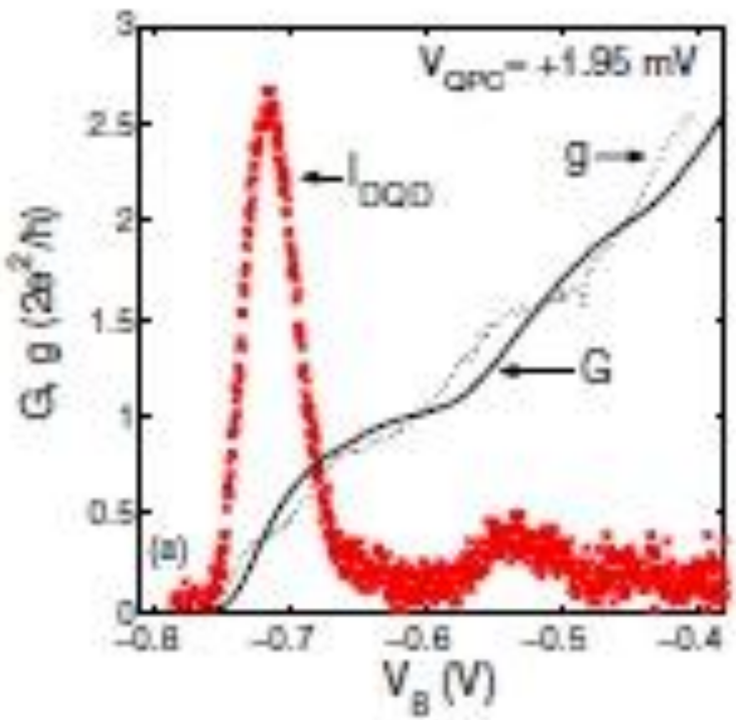
# Source and Detector

**E. Onac (Kouwenhoven) et al., PRL 96, 176601 (2006)**



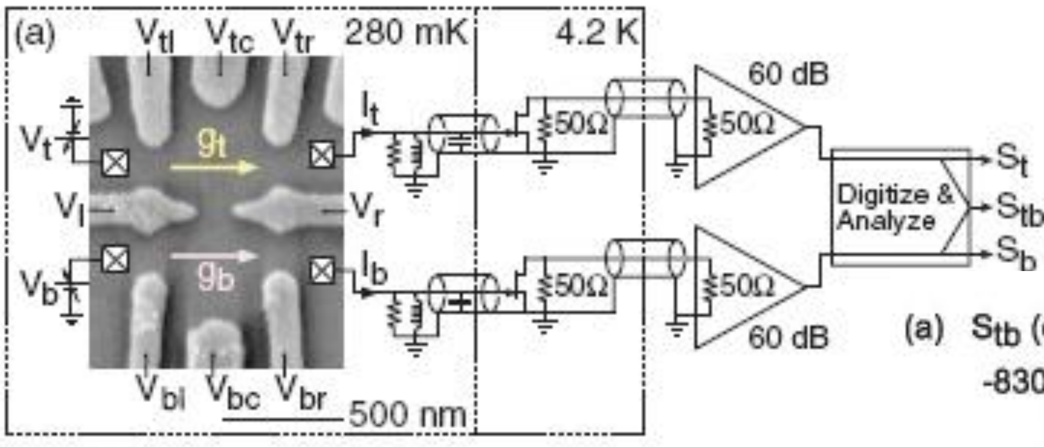

**V. S. Kharpai (Ludwig), PRL 97, 176803 (2006)**

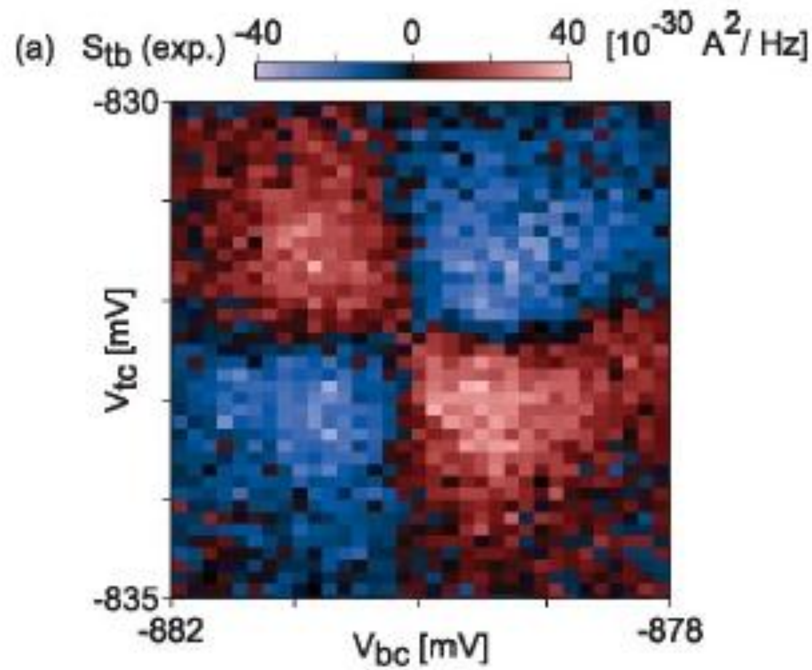


# Expriment on quantum dots

T. D. McClure et al., PRL 98, 056801 (2007)



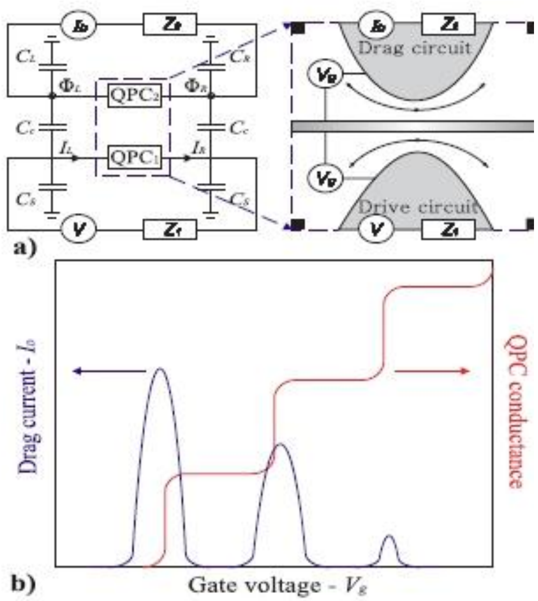

Current correlations

$$S_{12} = \int d\tau < \Delta I_1(\tau)\,\Delta I_2(0\,) >$$

M. C. Goorden and M. Buttiker, PRL 99; 146801 (2007); PRB 77, 205323 (2008)

# Noise induced transport

A. Levchenko and A. Kamenev, PRL 101, 216806 (2008)




- extrinsic coupling

- no internal dynamics

Drag current

$$I_D(V) = \int \frac{d\omega}{4\pi\omega^2} Tr[Z(\omega)S_1(\omega, V)Z(-\omega)\Gamma_2(\omega)]$$

Transimpedance matrix

$$Z_{\alpha\beta}(\omega) = d\Phi_\alpha / dI_\beta$$

Rectification in drag circuit

$$\Gamma_\alpha(\omega) = 2\frac{e^3}{h}(\hbar\omega)^2 \frac{dT_\alpha}{dE}$$

Excess noise in driver circuit (linear drag)

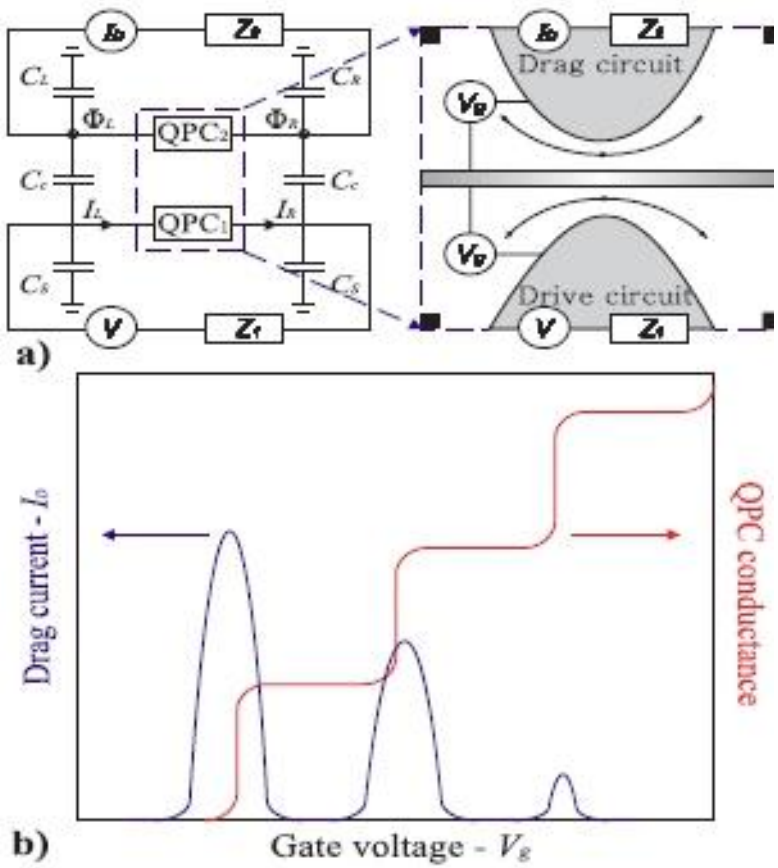
$$S_1(\omega, V) = V \frac{\partial}{\partial\omega}[coth\frac{\hbar\omega}{2kT}]\Gamma_1(\omega)$$

$$S_1(\omega, V) = 4\,kT\,G^{(2)}\,V$$

# Noise induced transport

A. Levchenko and A. Kamenev, PRL 101, 216806 (2008)

## Capacitively coupled quantum point contacts




External impedances

$$Z_1 << Z_2 << \frac{h}{e^2}$$

Linear drag (thermal noise)

$$I_D(V) = \frac{h}{e^2}\frac{\alpha\pi^2}{6}\frac{(kT)^2}{\Delta_1\Delta_2}\frac{V}{cosh^2(eV_g/2\Delta_1)}$$

Coupling constant $\alpha$

Non-linear drag (shot noise)

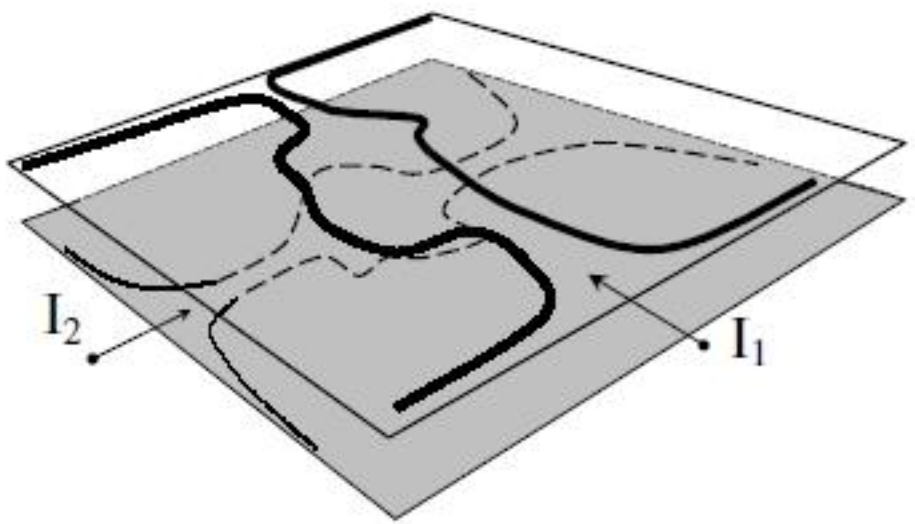
$$kT << eV << \Delta_1$$

$$I_D(V) = \frac{e^3V^2}{h}\frac{\alpha'}{\Delta_2}\sum_n T_n(1-T_n)$$

# Noise induced transport

A. Levchenko and A. Kamenev, PRL 101, 216806 (2008)

## Generic mesoscopic conductors




Conductance $g(E) = g + \delta g(E)$

Conductance fluctuations $\delta g(E)$

$g >> 1 \ , \ \delta g(E) \sim 1$

Thouless energy

$$E_T = \hbar D/L^2$$

Rectification

$$\Gamma(\omega) \sim \frac{e^3}{h}\frac{(\hbar\omega)^2}{E_T}$$

Drag current

$$I_D \sim \frac{e^2 V}{h}\left(\alpha\frac{(kT)^2}{E_T^2} + \alpha'\frac{eV}{E_T}g\right)$$

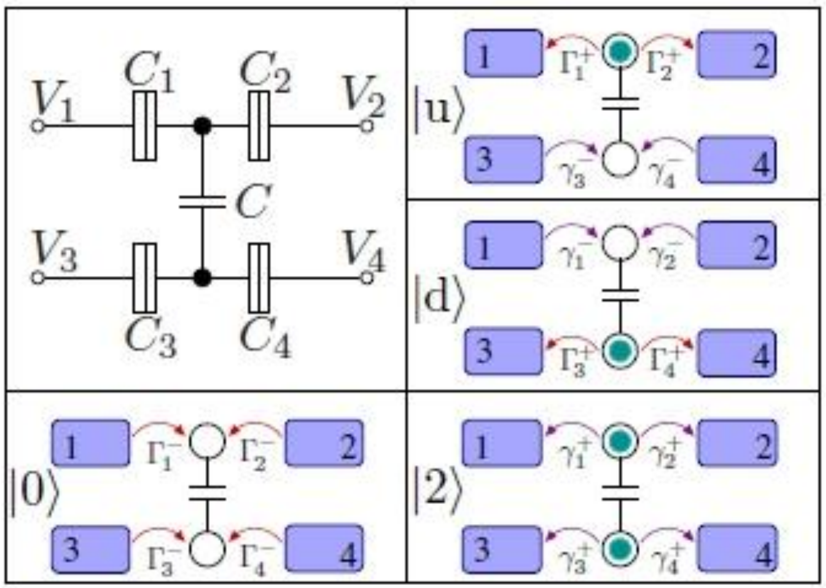
External versus internal coupling : here charging of conductor neglected

# (Shot)-Noise induced transport in quantum dots

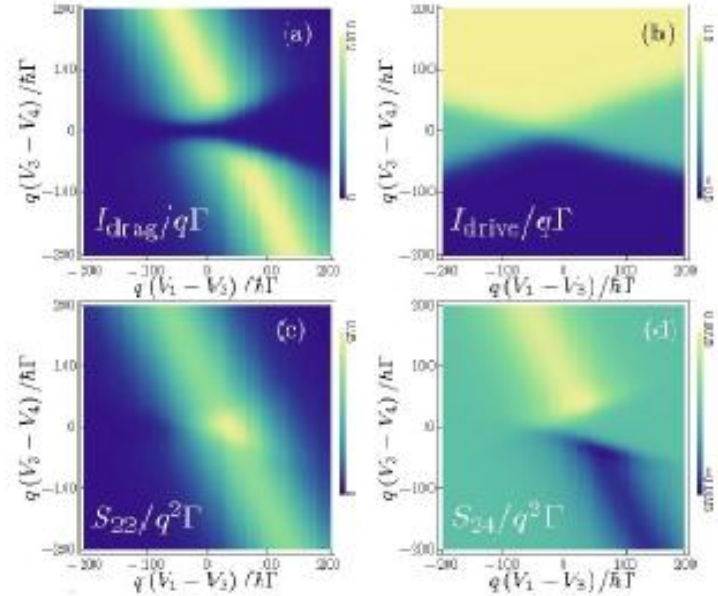
R. Sánchez, R.  López, D. Sánchez, and M. Büttiker , Phys. Rev. Lett. **104**, 076801 (2010)

$V_1 = V_2$

$V_3 > V_4$

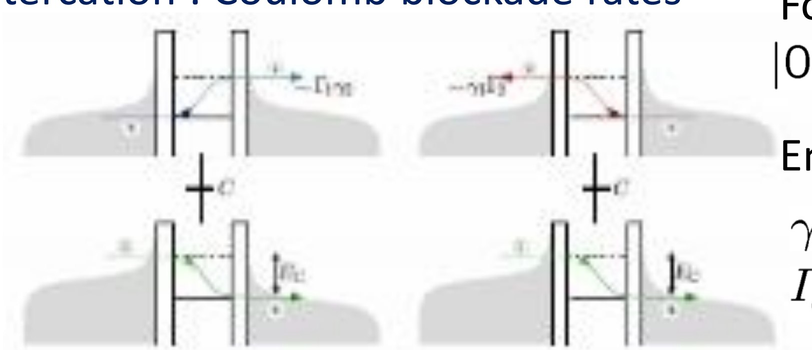



drive




gate

Realistic intercation : Coulomb blockade rates



Four states

$|0,0>, |0,1>, |1,0>, |1,1>$

Energy dependent rates

$\gamma_1 \Gamma_2 \neq \Gamma_1 \gamma_2$

$I_{drag} \approx \gamma_1 \Gamma_2 - \Gamma_1 \gamma_2$

Non-equilibrium noise (shot noise) of driven dot induces current in undriven dot

# Fluctuation relations

D. Andrieux and P. Gaspard, J. Stat. Mech. (2007) P 02006

H. Forster and M. Buttiker, PRL 101, 136805 (2008)

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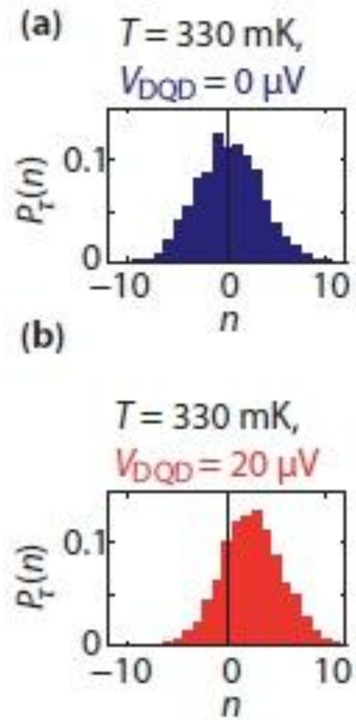



Multiprobe (equal temperature)

$$P(N_1, N_2, ...., t) = P(N, t)$$

Generating function

$$F(\chi) = ln \sum_N P(N,t) \exp(i\chi N)$$

Symmetry

$$F(i\chi) = F(-i\chi + qV/kT) \quad \Longrightarrow$$

$$P(N,t) = \exp(qNV/kT)\, P(-N,t)$$

Current

$$I_i = \sum_k G_{i,k} V_k + (1/2) \sum_{kl} G_{i,kl} V_k V_l + ......$$

Noise

$$S_{ij} = S_{ij}^{eq} + \sum_k S_{ij,k} V_k + ......$$

FDT

$$S_{ij}^{eq} = 2\, G_{i,j} kT + ......$$

Carrier exchanging terminals

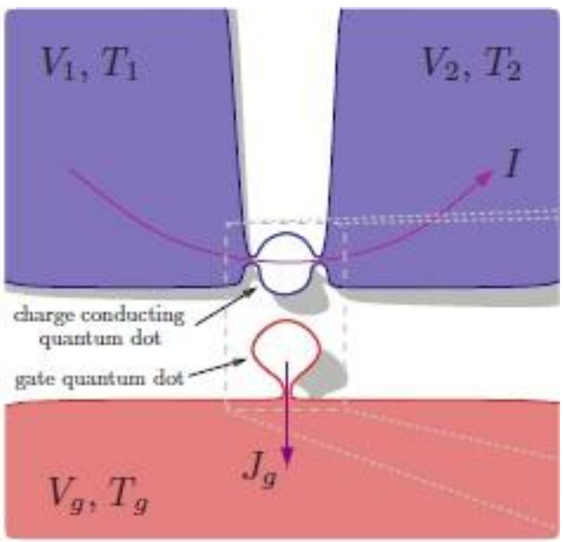
Gate voltage terminals

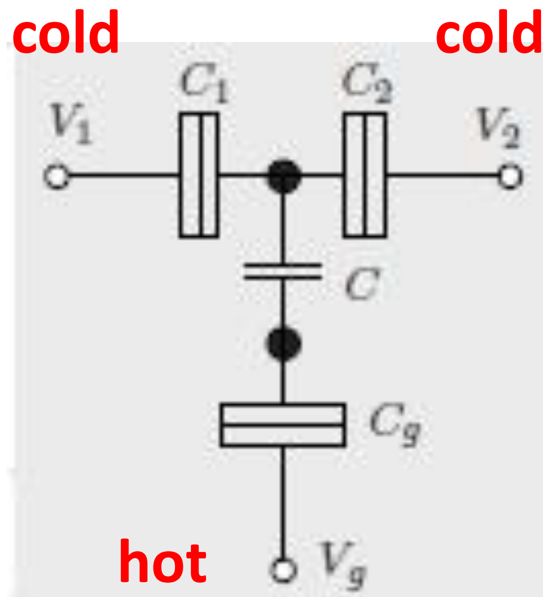
Noise susceptibility and rectification

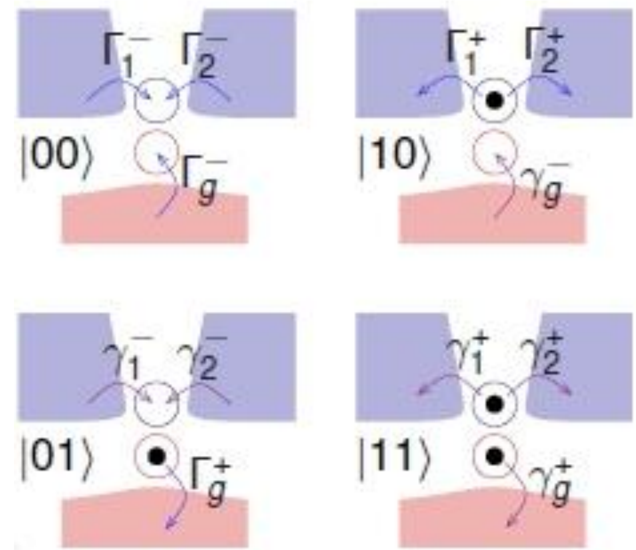
$$S_{ij,k} + S_{ik,j} + S_{jk,i} = kT\,(G_{i,jk} + G_{j,ik} + G_{k,ij})$$

# Thermal-noise induced current

R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)



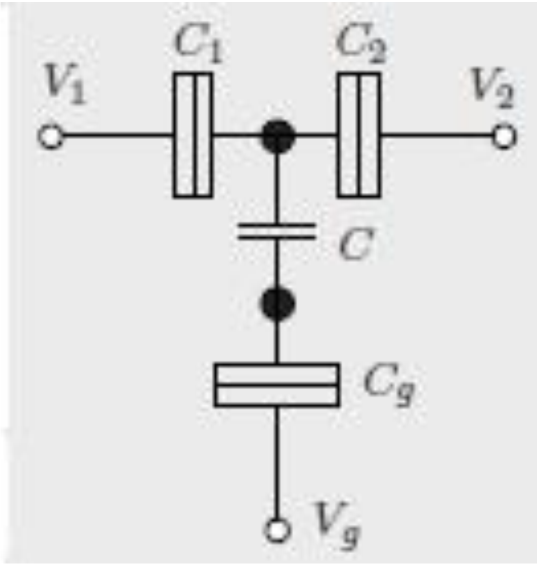



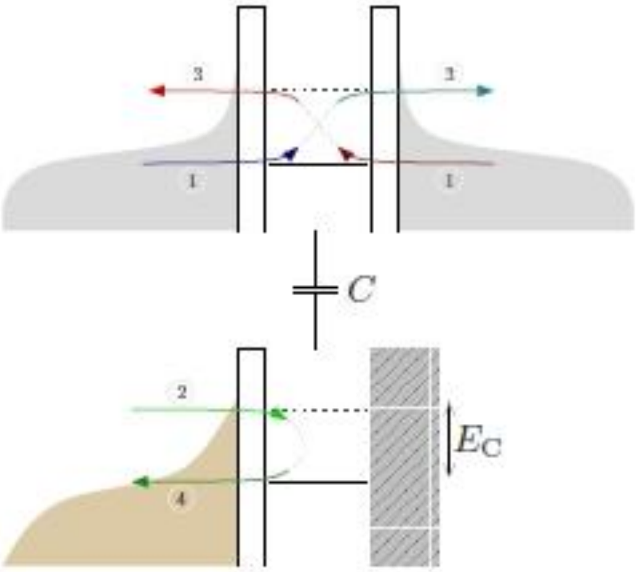

$$\Gamma_l^- = \Gamma_l\, f(\Delta_l^0/kT_l) \quad \Gamma_l^+ = \Gamma_l\,(1 - f(\Delta_l^0/kT_l))$$
$$\gamma_l^- = \gamma_l\, f(\Delta_l^1/kT_l) \quad \gamma_l^+ = \gamma_l\,(1 - f(\Delta_l^1/kT_l))$$

$$\Delta_l^n = \epsilon_u + U(1,n) - U(0,n) - qV_l$$
$$\Delta_g^n = \epsilon_d + U(n,1) - U(n,1) - qV_g$$
$$U(1,1) - U(0,1) = U(1,0) - U(0,0) + E_c \qquad E_c = \frac{2q^2C}{C_{\sum u}C_{\sum d} - C^2}$$

# Heat to charge conversion

R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)




$$V_1 = V_2$$

$$I = q\,\frac{\gamma_1\,\Gamma_2 - \Gamma_1\,\gamma_2}{(\Gamma_1 + \Gamma_2)\,(\gamma_1 + \gamma_2)}\,\frac{J_g}{E_c}$$

If $\gamma_1 = 0\,,\ \Gamma_2 = 0 \implies$

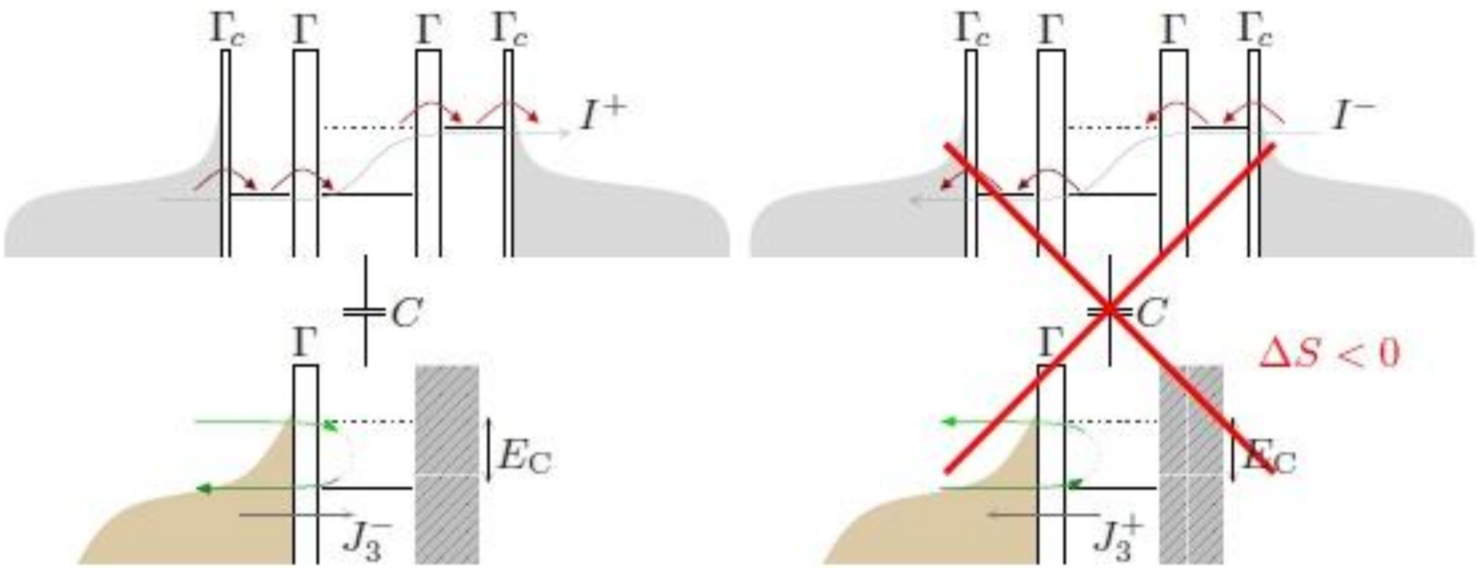
$$\frac{I}{q} = \frac{J_g}{E_c}$$

Every energy quantum of heat flow gets converted into a quantum of charge flow.

[T.E. Humphrey et al, Phys. Rev. Lett. 89, 116801 (2002)]

# Optimal converter geometry

R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)




Power against the potential: $P = I\,(V_1 - V_2)$ $\quad I = qJ_g/E_c$

Efficiency $\eta = P/(-J_g) = q\,(V_1 - V_2)/E_c$

Stopping potential $\Delta S = 0\,,\ I^+ = I^-\,,\quad \eta_C = 1 - T_s/T_g = q\Delta V_*/E_c$

(reversibility)

# Efficiency at maximum power

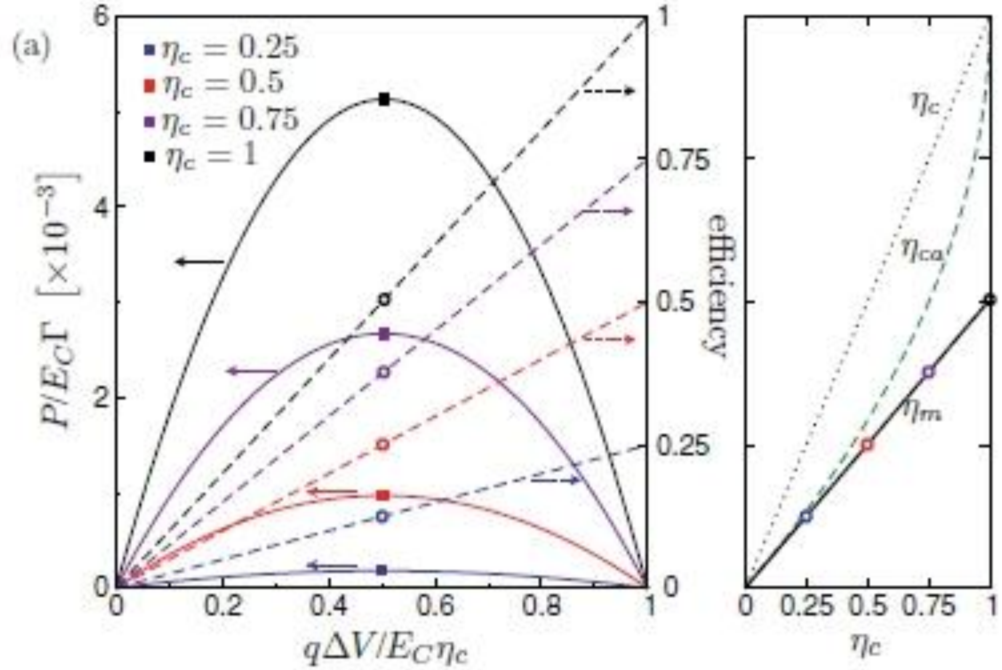
R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)

Carnot efficiency (no power) $\eta_C = 1 - T_s/T_g$

Curzon-Ahlborn efficiency $\eta_{ca} = 1 - \sqrt{T_s/T_g}$

Efficiency at maximum power $\eta_m$

Power of noise induced current as a function of load potential $\Delta V$
(units stopping potential)

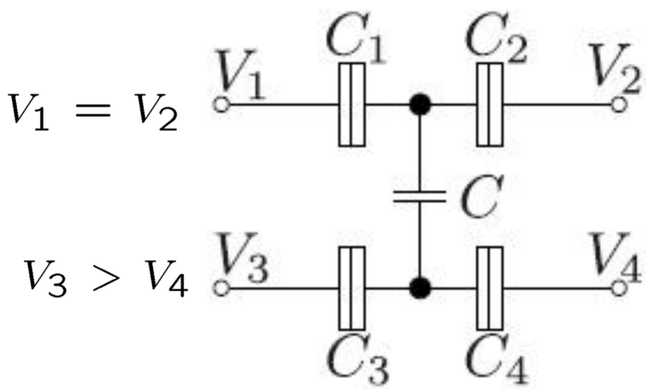



$$q\Delta V_*/E_c\eta_c = \Delta V/\Delta V_*$$

$$\eta = P/(-J_g) = q\,\Delta V/E_c = \eta_c\,\Delta V/\Delta V_*$$

# From four to three to two-probe structures

Four-probe :

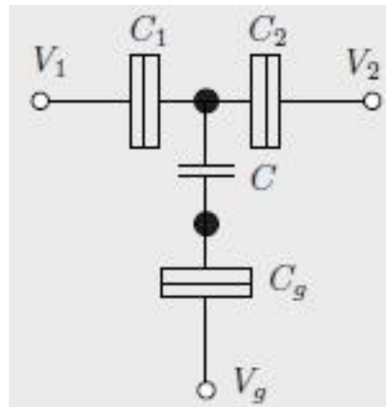



shot noise

$\Longrightarrow$

Electric currents

Three -probe:

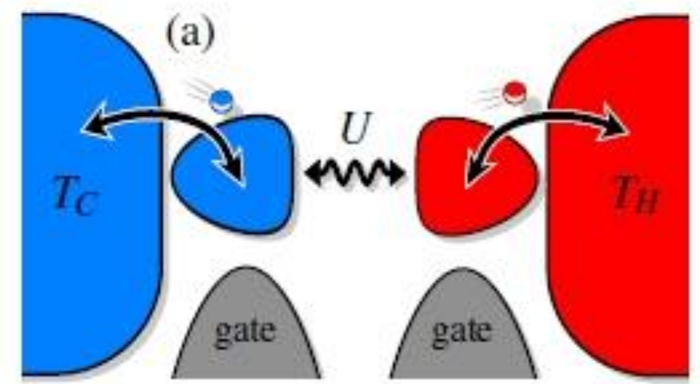



hot thermal noise

$\Longrightarrow$

Heat currents
Electric currents

Two-probe




hot thermal noise

$\Longrightarrow$

Heat currents (Diode)

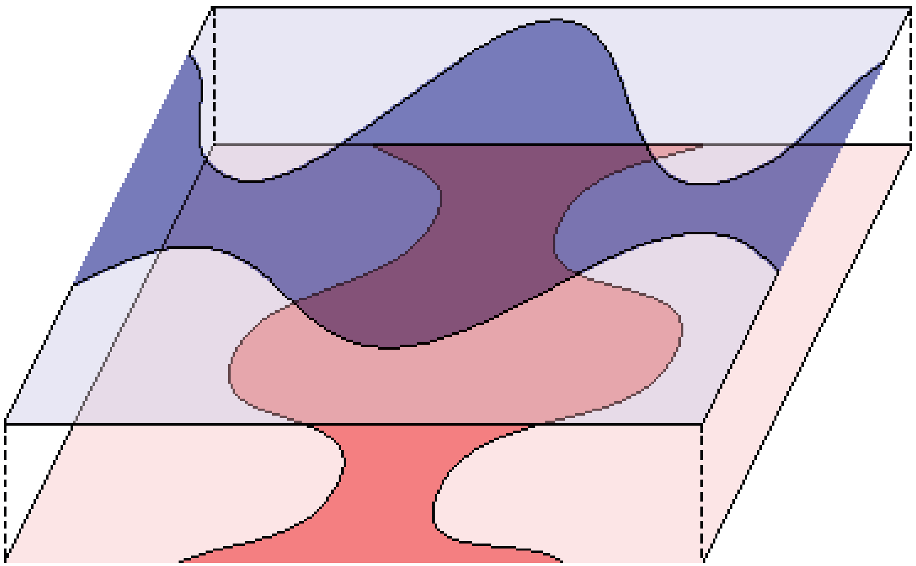
T.Ruokola
T. Ojanen,
PRB **83**, 241404 (2011)

# Current from hot spots: open dots

Bjorn Sothmann, Rafael Sanchez, Andrew Jordan, Markus Buttiker (unpublished)

Coulomb blockaded dots generate very small currents

Can we get larger currents in open dots?



Levchenko and Kamenv: Yes, but at best a single quantum channel effect.

Bjorn Sothmann et al (workshop): At a much higher energy scale exists there effects that lead to a much larger currents than predicted by mesoscopics?

# Summary

Current generated from non-equilibrium noise (hot spot)

Coulomb coupled quantum dots

Direction of heat current decoupoled from direction of charge current

Experimental tests possible in low temperature transport physics

Optimal heat to charge conversion

One energy quantum of the bath transfers one quantum of charge

System provides a goof test of non-equilibrium fluctuation relations