Cost of remembering a bit of information

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In 1961, Landauer [R. Landauer, IBM J. Res. Develop. **5**, 183 (1961)] pointed out that resetting a binary memory requires a minimum energy of $k_B T \ln(2)$. However, once written, any memory is doomed to lose its content if no action is taken. To avoid memory losses, a refresh procedure is periodically performed. We present a theoretical model and an experiment on a microelectromechanical system to evaluate the minimum energy required to preserve one bit of information over time. Two main conclusions are drawn: (i) in principle, the energetic cost to preserve information for a fixed time duration with a given error probability can be arbitrarily reduced if the refresh procedure is performed often enough, and (ii) the Heisenberg uncertainty principle sets an upper bound on the memory lifetime.

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The act of remembering is of fundamental importance in human life. Not only do manmade objects such as monuments and landscapes require maintenance to counterbalance their deterioration, but also biological systems are subject to the never-ending task of preserving shapes and functionalities by fighting the universal tendency of entropy to increase. As a consequence, the study of fundamental physical limits in memory devices [1] has received considerable attention in different contexts recently. Examples are in communication-theoretic paradigms [2], proteins functionality [3], biological noisy neural networks [4,5], future technologies [6,7], and in the presence of limited knowledge [8,9]. However, the fundamental energetic cost to preserve the state of a memory has received little attention so far. In this work, we investigate theoretically and experimentally the minimum energy cost required to preserve classical information stored in digital devices for a given time and with a given probability of failure.

To this end, we recollect that information is usually stored in digital devices through binary numbers (0 and 1). As a consequence, it is customary to represent a memory as a two-state physical system with an observable x and a bistable potential-energy landscape [Fig. 1(a)] [1,10–13]. The energy barrier allows one to define the two logic states, e.g., $x < 0$, representing bit 0, and $x > 0$, representing bit 1. Moreover, the barrier allows one to statistically confine x for a given time within one of the two wells [Fig. 1(b)], hence ensuring that one given bit is stored. This confined state is a nonequilibrium condition that evolves within the system relaxation time τ_k to thermal equilibrium [Fig. 1(f)]. This process is described via the time evolution of the probability density function $p(x,t)$ as follows. Let us assume we have a memory where bit 1 is stored. The initial probability density $p(x,0)$ shows a sharp peak centered in the right well [Fig. 1(b)]. According to the dynamic of the system, $p(x,t)$ will first relax inside the right well and then it will diffuse into the left well, thus developing a second peak [Figs. 1(b)–1(f)]. At any given time t, the probability that the system encodes the wrong logic state is represented by $P_0(t) = \int_{-\infty}^{0} p(x,t)dx$. Clearly, P_0 increases with time and reaches the thermal equilibrium condition $P_0 = 0.5$ when the memory is statistically lost [Fig. 1(f)].

To fight this natural deterioration of the bit, it is customary to perform a cyclic operation called *refresh*. This procedure consists in reading and then writing back the content of the memory, and it is periodically executed at intervals t_R [14,15]. The refresh operation restores a nonequilibrium condition by shrinking the width of each peak of $p(x,t)$. Note that during this refresh operation, no error correction is performed as the overall purpose is merely to fight the diffusive process leading to thermal equilibrium.

Based on this procedure, we can define the memory loss probability P_E at time $\bar{t}$, i.e., after $N = \bar{t}/t_R$ cycles, as

$$P_E = 1 - [1 - P_0(t_R)]^{\frac{\bar{t}}{t_R}}. \tag{1}$$

It indicates the probability to find the wrong value of the bit when the memory is interrogated at any time during the interval $[0 - \bar{t}]$ since its first writing, with a refresh interval t_R. In any practical application, it is interesting to *a priori* set both P_E and $\bar{t}$, and then deduce the optimal t_R to meet these targets. Assuming that the refresh operation has an energetic cost Q, what we want to address here is the fundamental minimum energetic cost Q_m to preserve a given bit for a time $\bar{t}$, with a probability of failure not larger than P_E, while executing the refresh procedure with periodicity t_R. To this end, we proceed as follows: we first investigate the maximum value of t_R for a given set of P_E and $\bar{t}$; second, we perform an experiment to measure the minimum energetic cost for a single refresh operation; finally, we estimate the physical fundamental limits associated with the overall procedure.

We start with the study of the maximum allowed value for t_R. Let us assume that the dynamics of the memory is characterized

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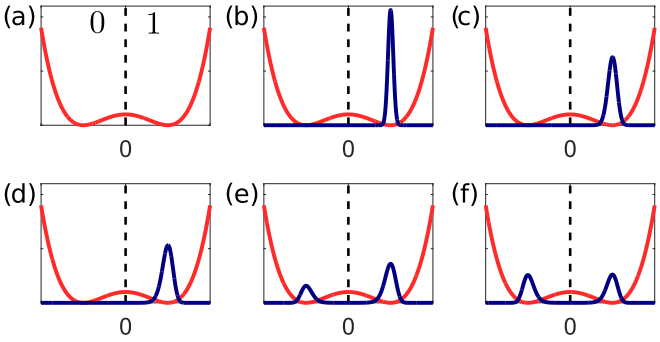

FIG. 1. (a) A generic binary memory is represented here in terms of the stochastic dynamics of a variable x subjected to a bistable potential (a). (b)–(f) The memory-loss mechanism when bit 1 is initially stored. Blue (dark gray) curves give a qualitative time evolution of $p(x,t)$ as the relaxation to equilibrium process takes place.

by a bistable Duffing potential,

$$U(x) = 4\left(-\frac{x^2}{2} + \frac{x^4}{4}\right). \tag{2}$$

The probability density function $p(x,t)$ thus evolves according to the following dimensionless Fokker-Plank equation [16,17]:

$$\frac{\partial}{\partial t}\, p(x,t) = \frac{\partial}{\partial x}\left[\frac{\partial U}{\partial x}\, p(x,t)\right] + T\,\frac{\partial^2}{\partial x^2}\, p(x,t), \tag{3}$$

where T is the temperature of the thermal bath. Solving numerically Eq. (3) and using Eq. (1), we obtain the maximum refreshing interval t_R that satisfies the *a priori* requirements for $\bar{t}$ and P_E (see Appendix A). Figure 2 shows the results of this study. We can see that large time $\bar{t}$ and small probability of error P_E yield short refresh time t_R, as expected.

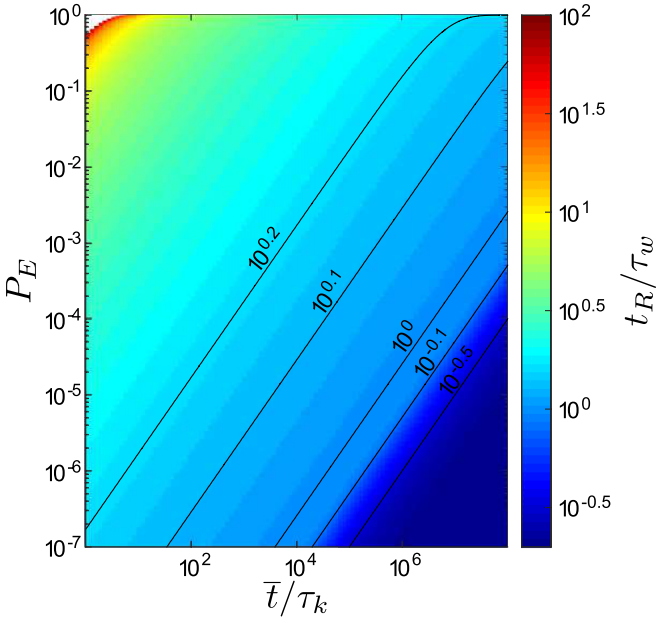

FIG. 2. Plot of t_R as a function of $\bar{t}$ and P_E for a memory modeled with bistable Duffing potentials. Here, t_R is given as a multiple of τ_w, i.e., the relaxation time of the harmonic approximation within one well.

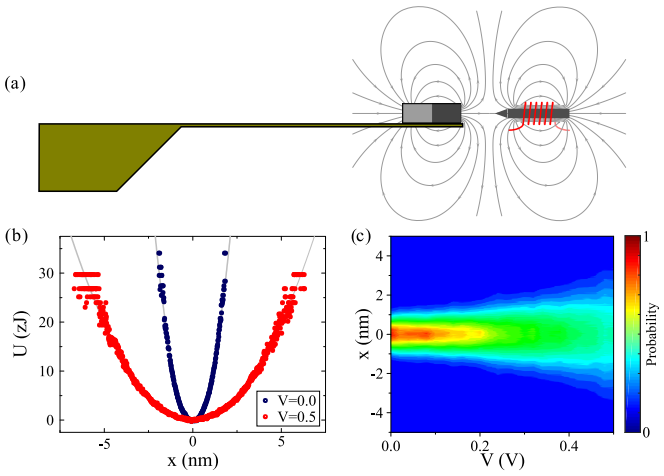

FIG. 3. Experimental setup. (a) Lateral view: a magnet on the cantilever tip and an electromagnetic coil are used to change the effective stiffness of the cantilever. (b) Potential energy of the cantilever tip, for two different voltages at the coil, reconstructed from the equilibrium probability density function. Solid gray lines represent the fitted harmonic potential. (c) Equilibrium probability density function of the cantilever tip position as a function of the voltage applied at the coil. The greater is the voltage, the greater is the repulsive force, resulting in a flattening of the potential and in a broadening of the equilibrium probability density function.

We now proceed to the second step of our program aimed at determining the minimum energetic cost for a single refresh operation. Within the formalism defined above, the refresh operation consists in shrinking $p(x,t_R)$ inside one of the wells of $U(x)$. Thus, the energetic cost becomes a function of t_R identified above. If we assume that $t_R \ll \tau_k$, the system dynamics is practically confined within one well. Here it can be approximately described by the dynamics of a harmonic oscillator, characterized by a Gaussian probability density function [18].

To estimate the energy cost associated with a real refresh procedure, we decided to perform an experiment employing a microelectromechanical oscillator composed of a 200-μm-long V-shaped structure with a nominal stiffness $k = 0.08\ \mathrm{Nm}^{-1}$, and a resonance frequency of 17 kHz. A tiny NdFeB (neodymium) magnet is attached to the cantilever tip with bicomponent epoxy resin reducing its resonance frequency to 5.3 kHz. An external electromagnet is placed in front of the cantilever, as depicted in Fig. 3(a). The deflection of the cantilever, x, is measured with an atomic force microscope (AFM)-like optical lever: a laser beam is focused on the cantilever tip with an optical lens (focal length $f = 50$ mm), and a small bend of the cantilever provokes the deflection of a laser beam that can be detected with a two-quadrant photodetector. For small cantilever deflections, the response of the photodetector remains linear, thus $x = r_x \Delta V_{PD}$, where ΔV_{PD} is the voltage difference generated by the two quadrants of the photodetector and r_x is a calibration factor obtained through the frequency response of the system under the action of thermal fluctuations. In the small-oscillation approximation,

the system dynamics can be modeled as a single degree of freedom subjected to a harmonic potential due to two forces: the cantilever restoring force and the magnetic force between the NdFeB magnet and the electromagnet. The measurement has been performed in vacuum, at pressure of 1×10^{-3} mbar. In this condition, the quality factor of the system is $Q_f = 300$, resulting in a relaxation time $t_{\rm relax} = 20$ ms. The experiment is conducted at room temperature and the system is subjected to thermal fluctuations and frictional forces as well. The magnetic force can be altered over time by varying the voltage on the electromagnet. In our experiment, the voltage applied to the coil results in a repulsive force with the effect of softening the potential energy of the system. The protocol used to perform the refresh operation is the following: at time $t = 0$, the voltage is linearly changed from the initial value $V = 0.5V$ to $V = 0V$. During this operation, the effective harmonic potential changes from the one represented by the red (light gray) dots in Fig. 3(b) to the blue (dark gray) ones. The equilibrium probability density function of the tip position changes accordingly, as depicted in Fig. 3(c), from right to left. The entire procedure takes a time t_p, after which the voltage on the coil is suddenly changed back to $V = 0.5V$ and kept in this condition for a time t_R.

The total work W performed by the external force on the memory system during the refresh operation can be estimated as [19,20]

$$W = \left\langle \int_0^{\tau_p} \frac{\partial H(x,V)}{\partial V}\dot{V}\,dt \right\rangle, \tag{4}$$

where $H(x,V)$ is the total energy of the system, $x(t)$ is the measured trajectory of the cantilever tip, $V(t)$ is the voltage applied on the electromagnet, and $\langle\cdot\rangle$ denotes the average over an ensemble of realizations. In particular, here we used ∼500 experimental trajectories for each selected time protocol τ_p under study. Since there is no variation on the internal energy of the system, the energetic cost Q of a refresh operation coincides with the work performed on the system ($Q = W$). This quantity has to be compared with the thermodynamic minimum $-T\Delta S$, where (see Appendix B)

$$\Delta S = k_B \ln\left(\frac{\sigma_i}{\sigma_f}\right) \tag{5}$$

is the entropy change associated with the refresh operation, σ_i is the target standard deviation of the Gaussian peak to be achieved with the refresh, and σ_f is the standard deviation of the Gaussian peak before the refresh. While σ_i can be arbitrary chosen, σ_f depends on t_R as (see Appendix B)

$$\sigma_f = \sqrt{\sigma_w^2 + \exp\left(-\frac{t_R}{\tau_w}\right)\left(\sigma_i^2 - \sigma_w^2\right)}, \tag{6}$$

where σ_w is the equilibrium standard deviation of the harmonic oscillator and τ_w is the relaxation time of the harmonic oscillator.

In Fig. 4(a), we show the measured values of Q required to perform a single refresh operation as a function of the protocol time t_p, for fixed σ_i and σ_f. We can see that Q approaches the minimum value given by Eq. (5) when t_p increases towards the quasistatic protocol condition. This observation is confirmed for different values of ΔS, as we can see from Fig. 4(b).

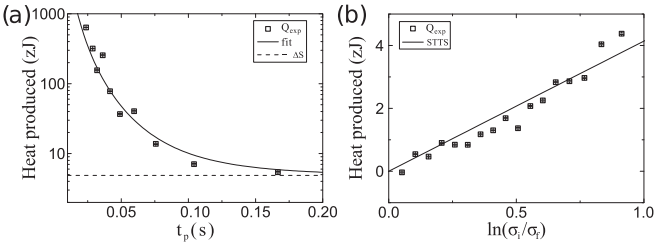

FIG. 4. Experimental results of produced heat. (a) Produced heat for a single refresh as a function of t_p. By increasing t_p, the produced heat tends to the lower bound, $Q = -T\Delta S$. Squares represent the heat from the experiment, while the solid line is the fit with the Zener dissipative model. (b) Produced heat in the quasistatic regime during a single refresh operation for different entropy variations. Squares represent the estimated heat from experiments, while the solid line is given by Eq. (5).

There we show the measured values of Q for a quasistatic protocol as a function of $-\ln(\sigma_i/\sigma_f)$. Experimental points are given as black squares, while the black solid line is the theoretical prediction from Eq. (5). As it is well apparent, the minimum energetic cost, represented by the thermodynamics bound $-T\Delta S$, can be reached in the quasistatic condition. The dissipative model behind the power-law fit in Fig. 4(a) is obtained by the Zener theory [13,21–24], assuming that the dissipative processes can be expressed as the result of frictional forces that represent the imaginary component of a complex elastic force $k(1 + i\phi)$. In general, ϕ is a function of the frequency and, for small damping, it can be expressed as the sum over all the dissipative contributions. In our case, $\phi(\nu) = \phi_{\rm str} + \phi_{th-el} + \phi_{\rm vis} + \phi_{\rm clamp}$. Here, $\phi_{\rm str}$ is the structural damping (ϕ is independent of the frequency ν), ϕ_{th-el} and $\phi_{\rm vis}$ are the thermoelastic and viscous damping, respectively, that can be assumed to be proportional to the frequency for frequencies much smaller than the cantilever characteristic frequency, and $\phi_{\rm clamp}$ represents the clamp recoil losses [$\phi(\nu) \propto \nu^3$].

Based on this result, we are now in a position to express the minimum fundamental cost Q_m for preserving a memory over a time $\bar{t}$ with a failure probability equal to P_E as

$$Q_m = -NT\Delta S = \frac{\bar{t}}{t_R}k_BT\ln\left(\frac{\sqrt{\sigma_w^2 + e^{-\frac{t_R}{\tau_w}}\left(\sigma_i^2 - \sigma_w^2\right)}}{\sigma_i}\right). \tag{7}$$

In Fig. 5(a), we show the minimum energy Q_m as a function of t_R for a given choice of P_E and $\bar{t}$. It is interesting to observe that this is an increasing function of t_R. In particular, Q_m approaches the value 0 when t_R goes to 0. This indicates that it is possible, at least in principle, to preserve the memory for a time $\bar{t}$ with failure probability P_E while spending zero energy. This is obtained when t_R approaches 0, but it also implies that the memory is always under refresh and never available for use. Moreover, Q_m diverges when t_R approaches a limit value $t_{R\rm Max}$ that depends on P_E. In fact, when $t_R \geqslant t_{R\rm Max}$, the imposed conditions on P_E and $\bar{t}$ cannot be satisfied. On approaching such a value, σ_i has to become smaller and smaller, thus requiring a larger and larger energy. This is apparent in Fig. 5(b), where we show the minimum energy Q_m

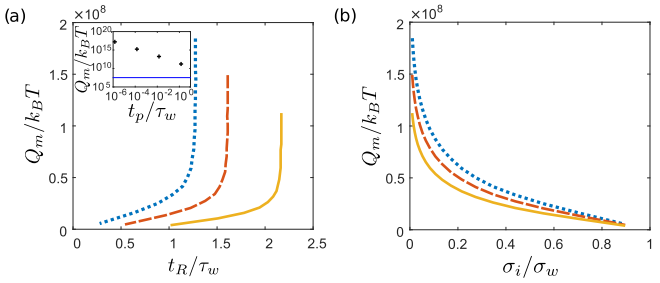

FIG. 5. Plots of Q_m to preserve the memory for $\bar{t} = 1\times10^3\tau_k$ as a function of (a) t_R and (b) σ_i. Blue (dotted) lines are obtained with $P_E = 1\times10^{-6}$, red (dashed) lines with $P_E = 1\times10^{-4}$, and yellow (solid) lines with $P_E = 1\times10^{-2}$. The inset in (a) shows the values for Q_m vs t_p/τ_w, when $\bar{t} = 1\times10^3\tau_k$, $P_E = 1\times10^{-6}$, and $t_R = \tau_w$. A finite protocol time t_p, which is typical of experiments, adds an excess dissipated heat to the blue line that marks the minimum value given by Eq. (7).

as function of σ_i for a given choice of P_E and $\bar{t}$. There, Q_m goes to 0 when σ_i goes to σ_w. This implies that it is indeed possible to preserve the memory for a time $\bar{t}$ with probability P_E by spending zero energy and this is realized when we operate extremely close to the equilibrium configuration inside one well ($\sigma_i \to \sigma_w$). On the contrary, as we anticipated, Q_m grows toward infinity when $\sigma_i \to 0$. Nonetheless, this last condition is limited by the Heisenberg uncertainty relation. By taking σ_i to coincide with the uncertainty on the position, we have $\sigma_i \geqslant \hbar/(2\sigma_p)$, where σ_p is the uncertainty on the momentum. This latter quantity, for a system at thermal equilibrium, can be estimated with the equipartition theorem if we have that k_BT is much greater than the energy separation between the system quantum levels. This is what happens in macroscopic devices that work at room temperature. Since the equipartition gives a finite value for σ_p, the uncertainty principle then sets a maximum accuracy on the position. This means that for a given system, the probability distribution of the relevant degree of freedom cannot be shrunk arbitrarily (see Appendix C) [25]. Since Q_m in Fig. 5(b) is a monotone function, we have that Q_m reaches a finite maximum value for the minimum allowed σ_i.

The existence of a minimum σ_i has a more important consequence: it sets a limit on our capability to preserve a given memory forever. This is apparent when we use Eq. (1) to explicitly write

$$\bar{t} = t_R \ln(1 - P_E)/\ln(1 - P_0). \tag{8}$$

Once we set P_E and select a finite t_R, we can make $\bar{t}$ as large as we want by properly selecting P_0 small enough. However, the existence of a finite minimum σ_i implies that P_0 can never be smaller than a nonzero minimum value, thus $\bar{t}$ reaches a finite maximum at best. To estimate such a maximum $\bar{t}$ in practical memories, we consider a micromechanical memory device such as the one in Ref. [12]. If we assume the distance between the two wells $x_m = 1\times10^{-9}$ m and a refresh period $t_R = 6.6\times10^{-3}$ s, we have that the minimum $\sigma_i = 9.6\times10^{-20}$ m. If we set $P_E = 1\times10^{-6}$, then the maximum value for $\bar{t}$ is approximately two years. On the other hand, if we set $P_E = 1\times10^{-4}$, then the maximum time $\bar{t}$ is approximately 200 years.

Finally, we briefly discuss the role of the protocol time t_p. As we have seen above from the experiment, the minimum fundamental bound Q_m can be reached only in the quasistatic regime where t_p is non-negligible. This condition sets a minimum value for t_R, such that $t_R \geqslant t_p$, and prevents the possibility to perform the experiment at zero energy expenditure. Moreover, for any finite t_p, frictional losses add to the minimum refresh cost Q_m, as is clearly visible from the experimental data in Fig. 4(a).

In order to identify a general estimate of the overall energy cost with a finite t_p, for a given choice of P_E, $\bar{t}$, and t_R, we use the formal tools developed in Refs. [11,26,27] to obtain a final condition of the protocol with the desired value of σ_i. The results are shown in the inset of Fig. 5(a). There we see that the dissipated energy Q_m is an inverse function of t_p, and that finite protocol times increase the energetic cost to refresh one bit by orders of magnitude with respect to the minimum cost prescribed by Eq. (7).

In conclusion, we studied the energy cost associated with memory preservation. We have introduced a physical model for the refresh procedure and realized an experiment in order to measure the amount of work performed during the refresh operation. Our study indicates that, in principle, we can preserve a digital memory for a given finite time with a given error probability while spending an arbitrarily little amount of energy. This is accomplished with refresh procedures that are performed arbitrarily often [Fig. 5(a)] and/or arbitrarily close to thermal equilibrium [Fig. 5(b)]. In practical cases, however, the existence of frictional forces introduces a lower limit on the refresh interval $t_R \geqslant t_p$ and this imply a nonzero minimum energy expenditure [Fig. 5(a), inset]. We have also shown that by the moment that the Heisenberg uncertainty principle implies the existence of a minimum width for the initial probability density of the memory device, any refresh strategy will inevitably fail after a finite time.

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APPENDIX A: COMPUTATION OF FIG. 2

To compute Fig. 2, we take

$$p(x,0) = \frac{\exp\left[-\frac{(x-1)^2}{2\sigma_i^2}\right]}{\sqrt{2\pi}\sigma_i} \tag{A1}$$

as the initial condition for Eq. (3). In particular, σ_i is such that $p(x,t)$ broadens inside the right well of $U(x)$ before developing a clean-cut second peak in the left well of the potential. We then solve Eq. (3) with the MATLAB pdepe function. With the solution, we compute

$$P_0(t_R) = \int_{-\infty}^{0} p(x,t_R) \tag{A2}$$

for different refresh times t_R, and then we evaluate the failure probability

$$P_E = 1 - [1 - P_0(t_R)]^{\frac{t}{t_R}} \tag{A3}$$

for different values of t_R and $t \gg t_R$. As a last step, we use a spline fit of P_E to sample t_R for different values of P_E and t. The results obtained in this way are plotted in Fig. 2. These results are obtained with $T = 1/8$, which corresponds to $\tau_k = 5.3\times10^4\tau_w$.

APPENDIX B: DERIVATION OF EQS. (5) AND (6)

Here we derive two important equations given in the main text, namely, Eqs. (5) and (6). We start with Eq. (6). To derive it, we assume that $T \ll 1$. This simplifies the mathematical description of the system as it implies that the intrawell relaxation mechanisms of the system are faster than the interwell ones. If we are interested in the intrawell mechanism only, then a satisfactory form for the dimensionless $p(x,t)$ is

$$p(x,t) = p_0(x,t) + p_1(x,t), \tag{B1a}$$

$$p_0(x,t) = P_0\frac{\exp\left[-\frac{(x+1)^2}{2\sigma(t)^2}\right]}{\sqrt{2\pi}\sigma(t)}, \tag{B1b}$$

$$p_1(x,t) = (1-P_0)\frac{\exp\left[-\frac{(x-1)^2}{2\sigma(t)^2}\right]}{\sqrt{2\pi}\sigma(t)}, \tag{B1c}$$

where P_0 is, to all effects, constant over time. We substitute Eq. (B1) in Eq. (3) and then we approximate Eq. (2) with a harmonic potential by Taylor expanding around $x = \pm1$. This yields two distinct equations,

$$\frac{\partial p_0}{\partial t} - 8\left[p_0 + (x+1)\frac{\partial p_0}{\partial x}\right] - T\frac{\partial^2 p_0}{\partial x^2} = 0, \tag{B2a}$$

$$\frac{\partial p_1}{\partial t} - 8\left[p_1 + (x-1)\frac{\partial p_1}{\partial x}\right] - T\frac{\partial^2 p_1}{\partial x^2} = 0, \tag{B2b}$$

where we used the fact that $p_1(x,t)$ $[p_0(x,t)]$ cannot affect the dynamics of the system in the left [right] well of $U(x)$ if $\Delta U \ll k_BT$. Equation (B2) can be combined into

$$\int_{-\infty}^{\infty}\left\{\frac{\partial p_0}{\partial t} - 8\left[p_0 + (x+1)\frac{\partial p_0}{\partial x}\right] - T\frac{\partial^2 p_0}{\partial x^2}\right\}(x+1)^2dx, + \int_{-\infty}^{\infty}\left\{\frac{\partial p_1}{\partial t} - 8\left[p_1 + (x-1)\frac{\partial p_1}{\partial x}\right] - T\frac{\partial^2 p_1}{\partial x^2}\right\}(x-1)^2dx = 0, \tag{B3}$$

which reduces to

$$\frac{\partial\sigma(t)^2}{\partial t} + 16\sigma(t)^2 - 2T = 0. \tag{B4}$$

Equation (B4) describes the time evolution of $\sigma(t)$ when intrawell relaxation mechanisms occur. Its analytic solution for an initial condition $\sigma(0) = \sigma_i$ is

$$\sigma(t) = \sqrt{\frac{T}{8} + \exp(-16t)\left(\sigma_i^2 - \frac{T}{8}\right)}, \tag{B5}$$

which is the dimensionless version of Eq. (6) given in the main text.

To compute Eq. (5), we recollect that we stated that the "refresh operation restores a nonequilibrium condition by shrinking the width of each peak of $p(x,t)$," without error corrections. If we assume that the refresh protocol preserves the symmetry of $U(x)$, then $p(x,t)$ can be written as Eq. (B1) during the whole refresh procedure. As a consequence, the sole effect of a refresh operation with duration t_p is to transform

$$p(x,t) = P_0(t)\frac{\exp\left[-\frac{(x+1)^2}{2\sigma(t)^2}\right]}{\sqrt{2\pi}\sigma(t)} + [1-P_0(t)]\frac{\exp\left[-\frac{(x-1)^2}{2\sigma(t)^2}\right]}{\sqrt{2\pi}\sigma(t)} \tag{B6}$$

into

$$p(x,t+t_p) = P_0(t)\frac{\exp\left[-\frac{(x+1)^2}{2\sigma_i^2}\right]}{\sqrt{2\pi}\sigma_i} + [1-P_0(t)]\frac{\exp\left[-\frac{(x-1)^2}{2\sigma_i^2}\right]}{\sqrt{2\pi}\sigma_i}, \tag{B7}$$

where $\sigma(t)$ is given by Eq. (B5), $P_0(t)$ is fitted from the numerical solution of Eq. (3) with Eq. (2), and $\sigma_i = \sigma(0)$. We now use the Gibbs entropy definition,

$$S(t) = -k_B\int_{-\infty}^{\infty} p(x,t)\ln p(x,t)dx, \tag{B8}$$

to compute the entropy variation $\Delta S = S(t+t_p) - S(t)$ of the refresh protocol. Because of the $\Delta U \gg k_BT$ assumption, we have that

$$\Delta S \approx -k_B\left\{\int_{-\infty}^{\infty}\frac{e^{-\frac{x^2}{2\sigma_i^2}}}{\sqrt{2\pi}\sigma_i}\ln\left(\frac{e^{-\frac{x^2}{2\sigma_i^2}}}{\sqrt{2\pi}\sigma_i}\right)dx - \int_{-\infty}^{\infty}\frac{e^{-\frac{x^2}{2\sigma(t)^2}}}{\sqrt{2\pi}\sigma(t)}\ln\left[\frac{e^{-\frac{x^2}{2\sigma(t)^2}}}{\sqrt{2\pi}\sigma(t)}\right]dx\right\}, \tag{B9}$$

which reduces to

$$\Delta S \approx k_B\ln\left[\frac{\sigma_i}{\sigma(t)}\right]. \tag{B10}$$

By using Eq. (B10) with $t = t_R$, we obtain Eq. (5) presented in the main text.

APPENDIX C: MINIMUM VALUE FOR σ_i

We discuss here the existence of the minimum possible value for σ_i. First of all, we observe that $\sigma_i \to 0$ is a singular limit in Eq. (B10). This is inconsistent with the third law of thermodynamics, so there must be a minimum value for σ_i. This is given by the Heisenberg uncertainty principle. In the best-case scenario, this reads

$$\sigma_x\sigma_p = \frac{\hbar}{2}, \tag{C1}$$

where σ_x (σ_p) is the uncertainty on the position x (momentum p). According to the equipartition theorem,

$$\sigma_p = m\sqrt{\langle v^2\rangle - \langle v\rangle^2} = \sqrt{mk_BT}, \tag{C2}$$

so we have that

$$\sigma_x = \frac{\hbar}{2\sqrt{mk_BT}}. \tag{C3}$$

Equation (C3) sets the minimum possible uncertainty for σ_x. Since σ_i describes the uncertainty of the initial x value, we therefore have that $\sigma_i \geqslant \sigma_{i\mathrm{Min}} = \frac{\hbar}{2\sqrt{mk_BT}}$. The existence of a $\sigma_{i\mathrm{Min}}$ implies that even at $t = 0$, the probability of error P_0 is greater than zero. Clearly, this does not exclude that one can have a smaller σ_i by accepting a larger σ_p. This would imply to operate the memory out of the thermal equilibrium, growing the dissipated energy well above the fundamental minimum.

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