Elogio del rumore

dalla
Risonanza Stocastica
al
Energy Harvesting

Luca Gammaitoni

NiPS Laboratory, Dipartimento di Fisica

Università degli Studi di Perugia and INFN Perugia, Italy

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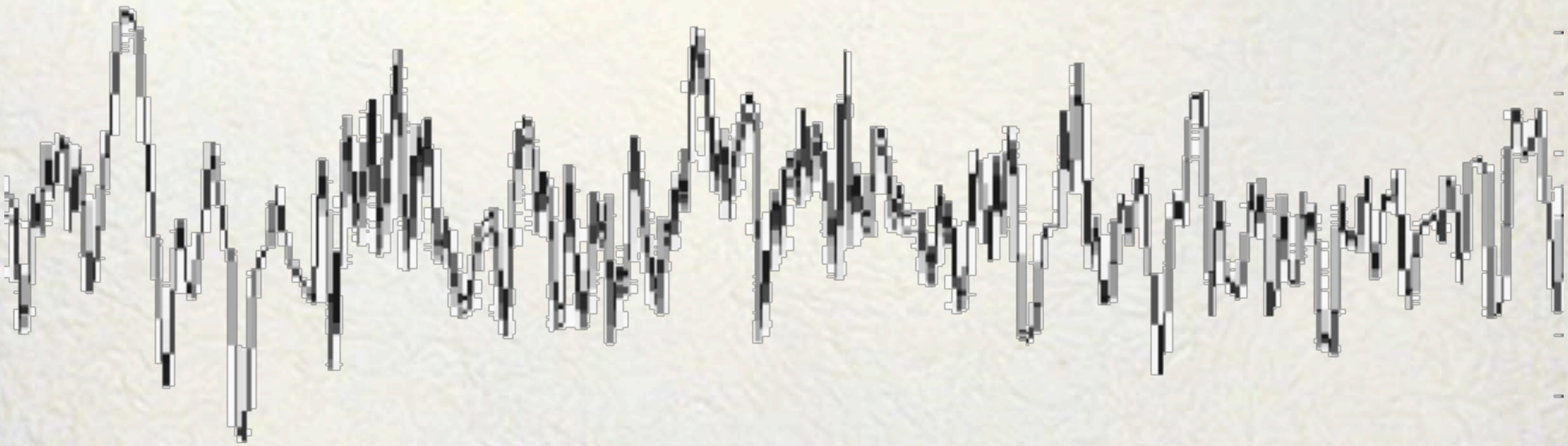
Noise in Physical Systems

La metta così:
dove niente sta al posto giusto, c'è disordine.
Dove al posto giusto non c'è niente, lì c'è ordine

B. Brecht, Dialoghi di profughi



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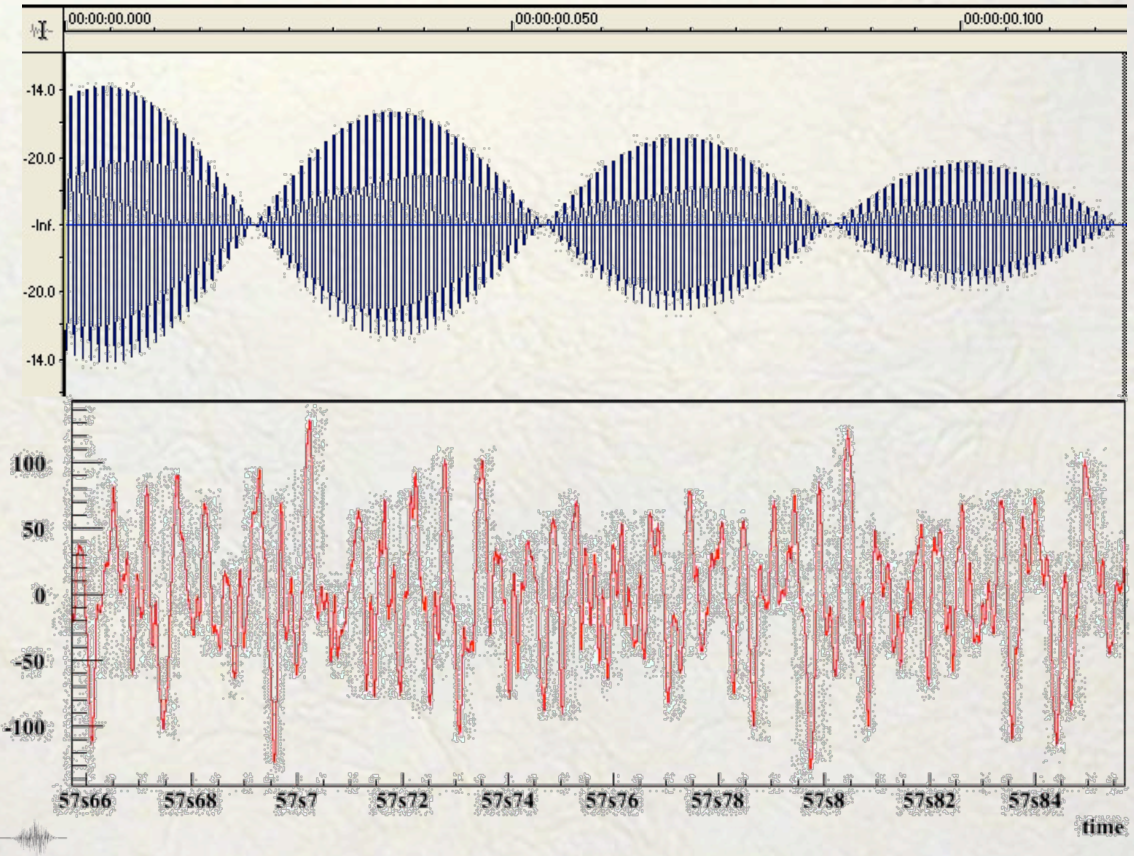

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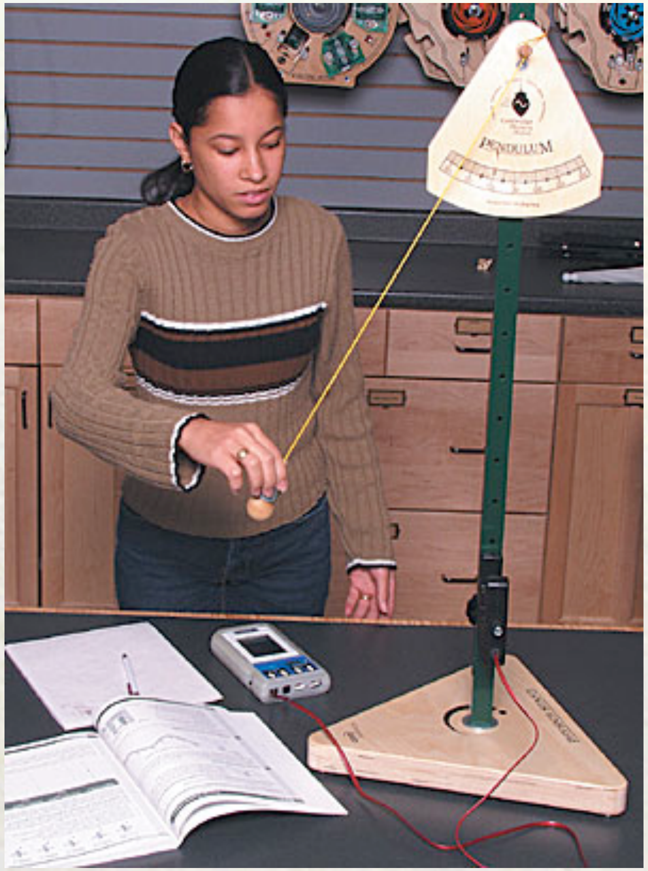
What do we mean with noise ?

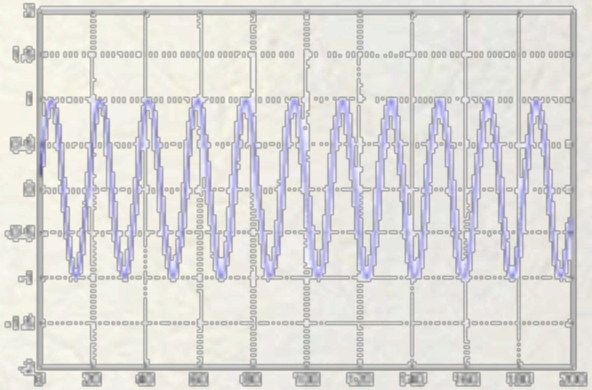
Noise in Physics means:
A disturbance, especially a random and persistent disturbance, that obscures or reduces the clarity of a signal.

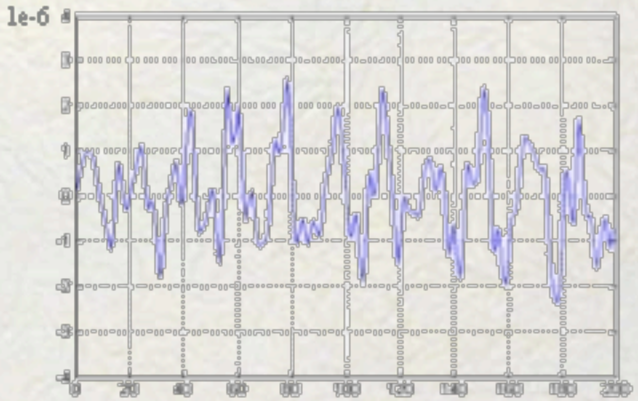
Noise becomes relevant when we try to measure a physical quantity with high **precision**

“**Precision** is the ability of an instrument to produce the same value or result, given the same input”

Example: the measurement of the pendulum position

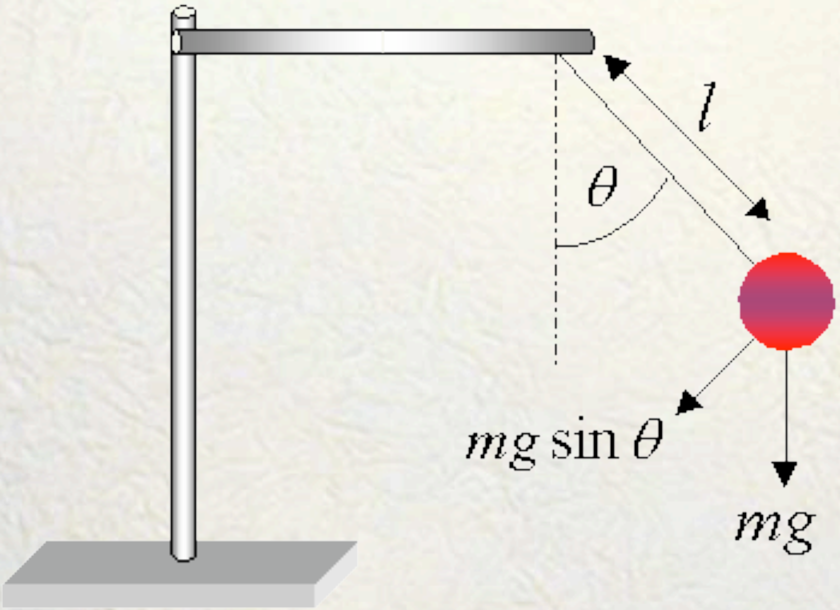






Where does this noise comes from?

The almost "simple" pendulum



Motion equation

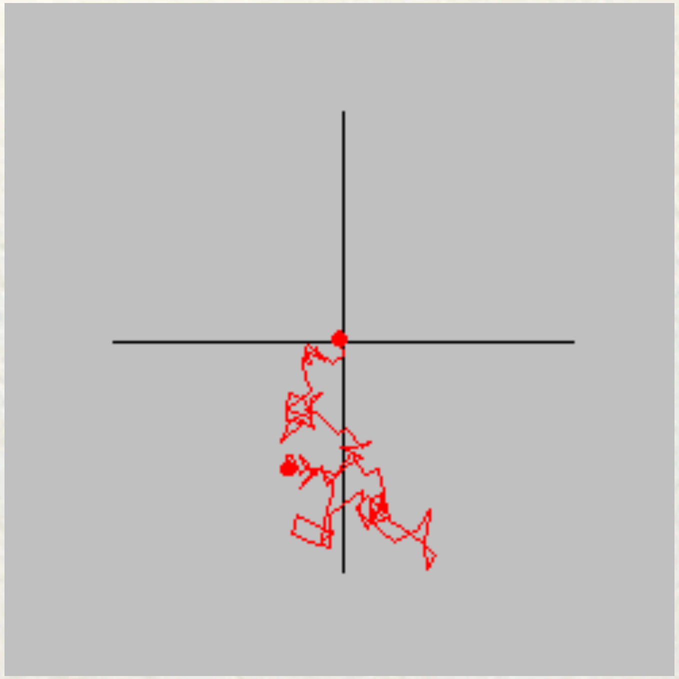
$$ml^2\frac{d^2\theta}{dt^2}+mgl\sin\theta = 0$$

Small oscilation approximation, with damping

$$\frac{d^2\theta}{dt^2}+\left(\frac{1}{q}\right)\frac{d\theta}{dt}+\omega^2\sin\theta = 0$$

What if we wait long enough ?

The **very small** oscillation limit.
Let the pendulm swing freely…

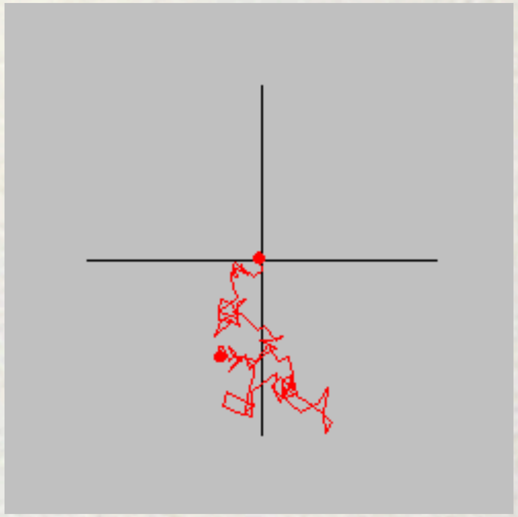


Example: the real measurement of a free swinging pendulum

Mass m= 1 Kg
Length l = 1 m
rms motion = 2 10^{-11} m

Mass m= 1 g
Length l = 1 m
rms motion = 6 10^{-10} m

Mass m= 10^{-6} g
Length l = 1 m
rms motion approx 1 mircon



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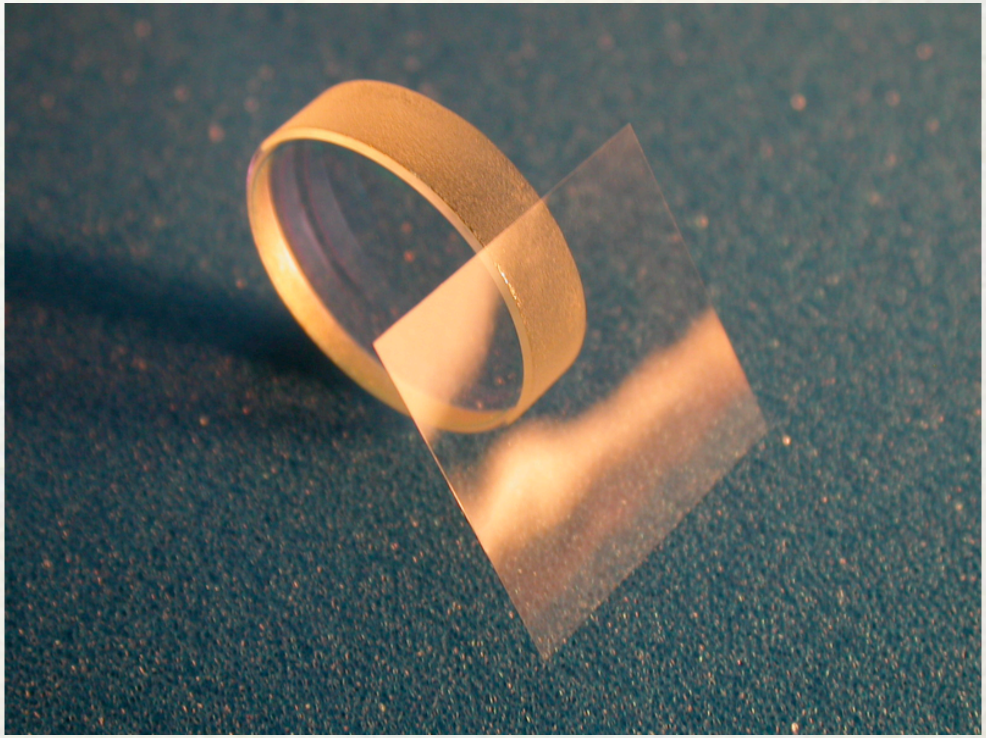
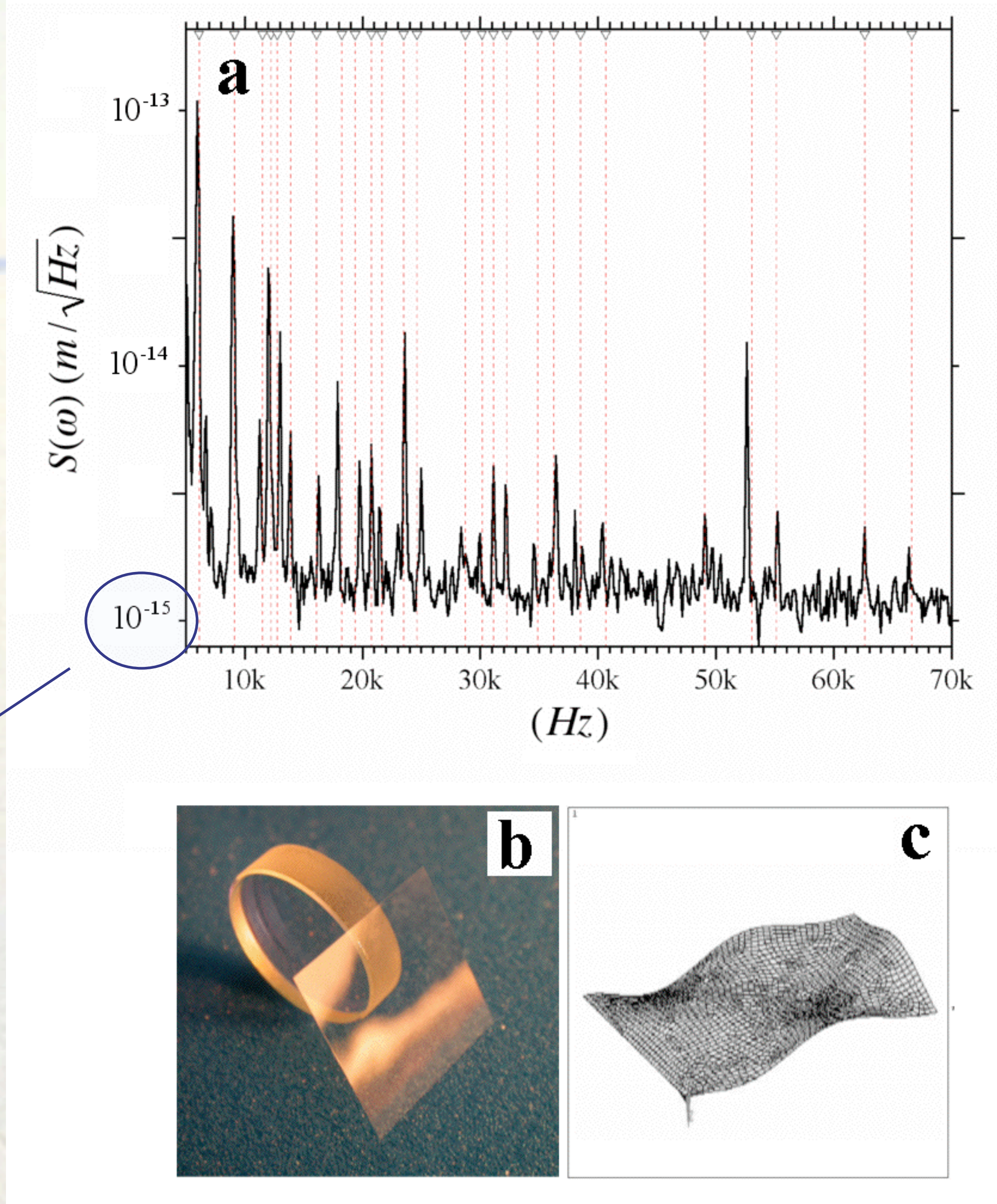
Direct measurement of internal vibration on a thin fused silica slab



Risultati sperimentali



10^{-15}

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The mother of all noises: **thermal noise**

Brownian motion → thermal noise

thermal noise is the name commonly given to **fluctuations** affecting a physical observable of a macroscopic system at **thermal equilibrium** with its environment.

The **internal energy** of a macroscopic apparatus at thermal equilibrium is <u>shared between all its degrees of freedom</u> or, equivalently, <u>between all its normal modes</u> each carrying an average energy **kT**, where T is the equilibrium temperature.

This is true also for such modes as the oscillations of springs, pendula, needles, etc.

Such an **energy** manifests itself as a **random fluctuation** of the relevant observable experimentally perceived as the noise affecting its measured value.

How to model **thermal noise**

To describe such a dynamics it is necessary to introduce a statistical approach

a) Fokker-Plank equation

b) Langevin Equation

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The usual equation of motion (Newton f = ma) should be changed in order to accomodate non deterministic forces. What we get is called **Langevin equation**:

The are two kind of forces acting on the pollen grain:

1) A viscous drag (motion in a fluid) $-6\pi\,\eta\,a\,\dot{x}$

2) A fluctuating force (representing the incessant impact of the molecules) $\zeta(t)$

$$m\,\ddot{x} = -6\pi\,\eta\,a\,\dot{x} + \zeta(t)$$

Generalization:

The viscous drag expression can be generalized in order to describe a wider class of damping functions

$$-m\int_{-\infty}^{t}\gamma(t-\tau)\,\dot{x}\,d\tau$$

Generalized Langevin equation

$$m\,\ddot{x} = -m\int_{-\infty}^{t}\gamma(t-\tau)\,\dot{x}\,d\tau + \zeta(t)$$

$\zeta(t)$ Is the stochastic force with known statistical properties:

Probability density function, moments, correlations, ...

For the pendulum we get:

$$\ddot{x} = -\omega_p^2\, x - \int_{-\infty}^{t} \gamma(t-\tau)\,\dot{x}\, d\tau + \frac{\zeta(t)}{m}$$

The arbitrariness in our choice of the damping function and the stochastic force is constrained by the so-called Fluctuation-Dissipation theorem which suffices to enforce the energy equipartition

$$\langle \zeta(t)\zeta(0)\rangle = k\,T\, m\gamma(|t|)$$

In the spectral domain, for a linear system, is always possible to write its response to an external force like:

$$X(\omega) = H(\omega)F(\omega)$$

Where H is the system transfer function.

$$H(\omega) = H'(\omega) + i\,H''(\omega) = \left|H(\omega)\right| e^{i\phi(\omega)}$$

The F-D Theorem can be written here as:

$$S_x(\omega) = -4\,k\,T\,\frac{H''(\omega)}{\omega}$$

The dissipative properties of the dynamical system are thus directly related to the equilibrium fluctuations.

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Fluctuation-Dissipation theorem

The **dissipative properties** of the dynamical system are thus directly related to the equilibrium fluctuations.

Physical connection:

the source of the fluctuations is the very same of the source of the dissipation

Why should we bother with thermal noise ?

Physics is like sex.
Sure, it may give some practical results,
but that's not why we do it.

Richard Feynman

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Cambio di prospettiva

Anziché considerare il rumore un mero disturbo, ipotizzare un suo impiego costruttivo.

Due esempi:

1) La Risonanza Stocastica
2) Effetto Wisepower

Dynamical description

For the generic physical dynamic system we can use the "potential" description (conservative forces):

$$\ddot{x} = -V'(x) - f(\dot{x}) + \zeta(t)$$

Where we set m=1 and
introduced the dissipation function *f*

For the linear pendulum = harmonic oscillator

$$V(x) = \frac{1}{2} m \omega_p^2 x^2$$

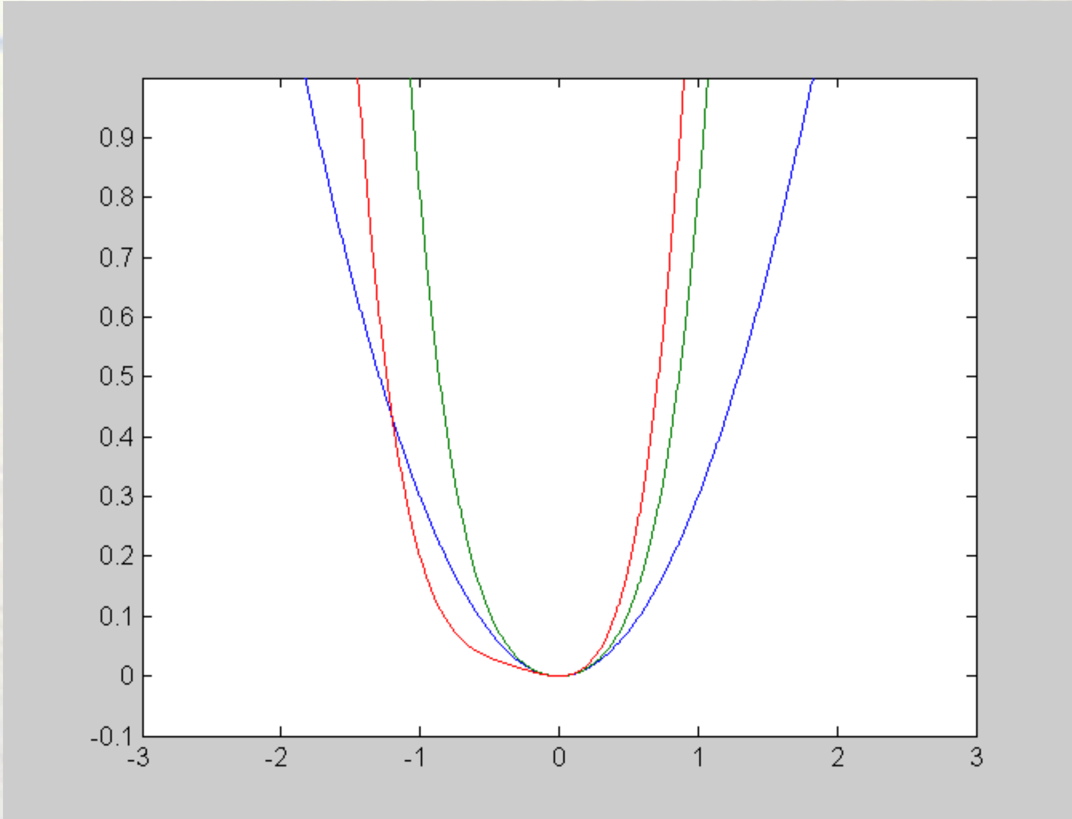
The potential of a linear system is a parabolic one!

Langevin Equation for the linear pendulum

$$\ddot{x} = -\omega_p^2 \, x - \int_{-\infty}^{t} \gamma(t-\tau) \, \dot{x} \, d\tau + \frac{\zeta(t)}{m}$$

nonlinear dynamics in **nonlinear** potential

We can change the shape of the curve by adding terms to the polinomial expression of the potential



Monostable nonlinear potential

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Bistable (nonlinear) potential

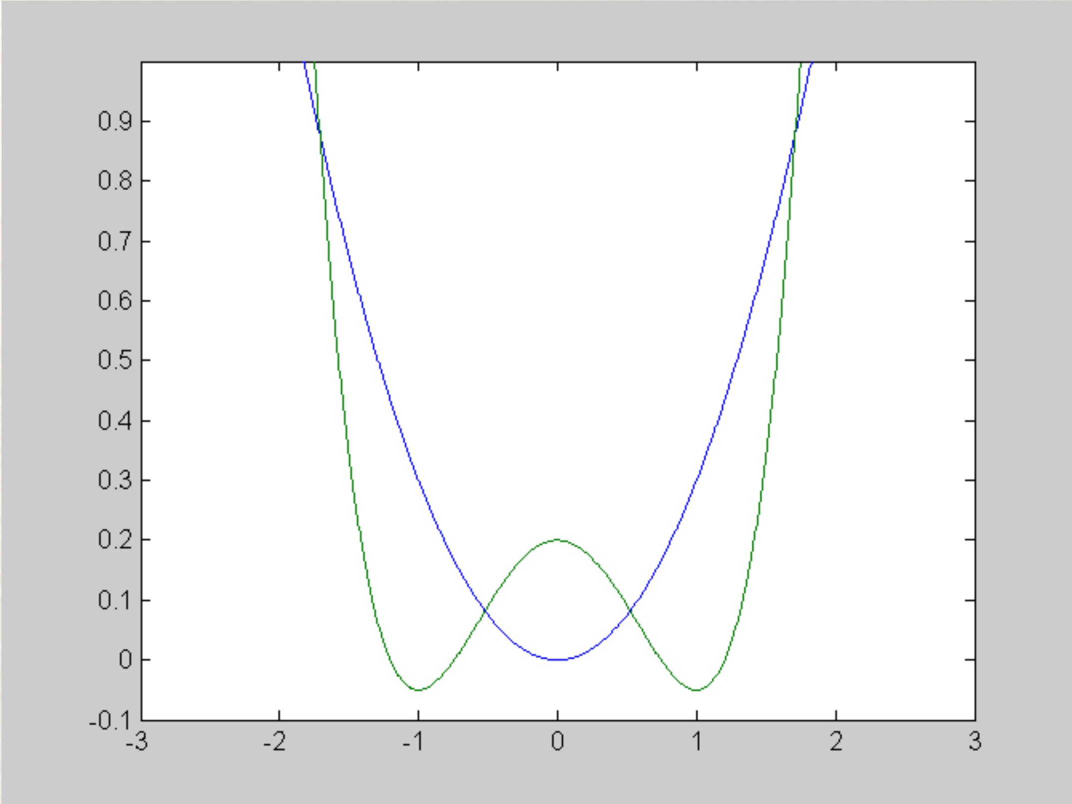
$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$

Bistable …

Add cubic term

Add quartic term

symmetric
quartic



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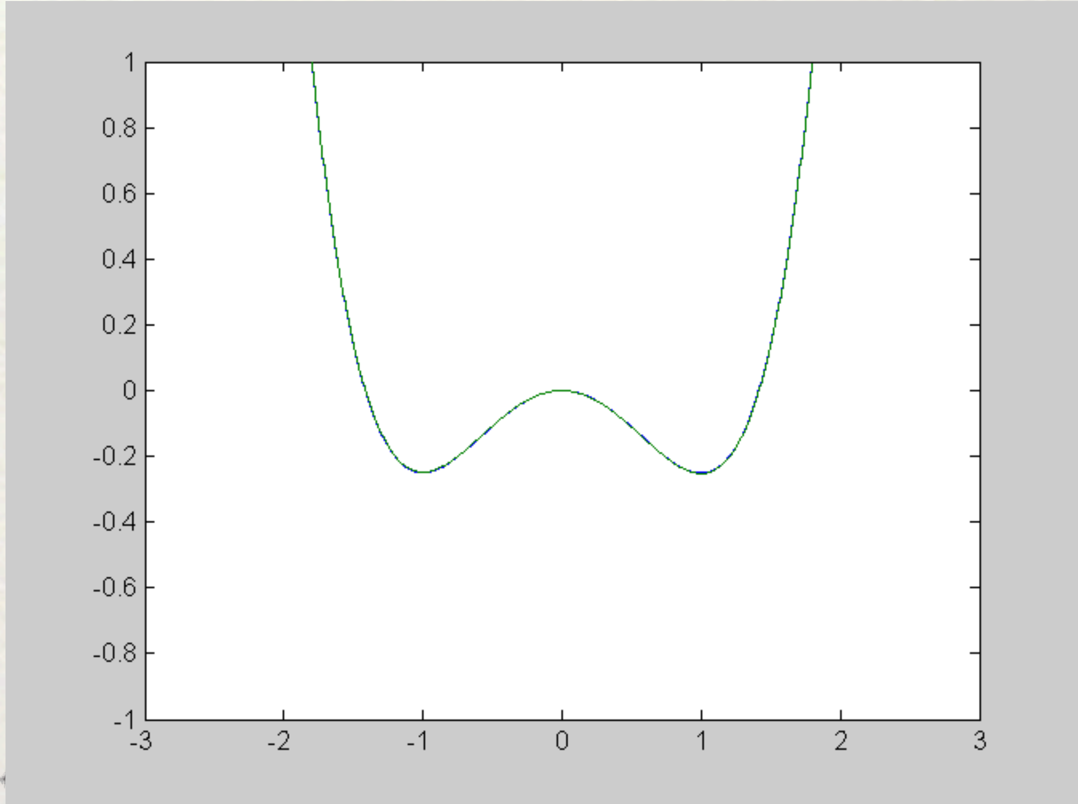
What if …

I introduce the time dependent part into the potential ?

$$V(x,t) = V(x) + A(t) \qquad A(t) = A\sin(\omega t)$$



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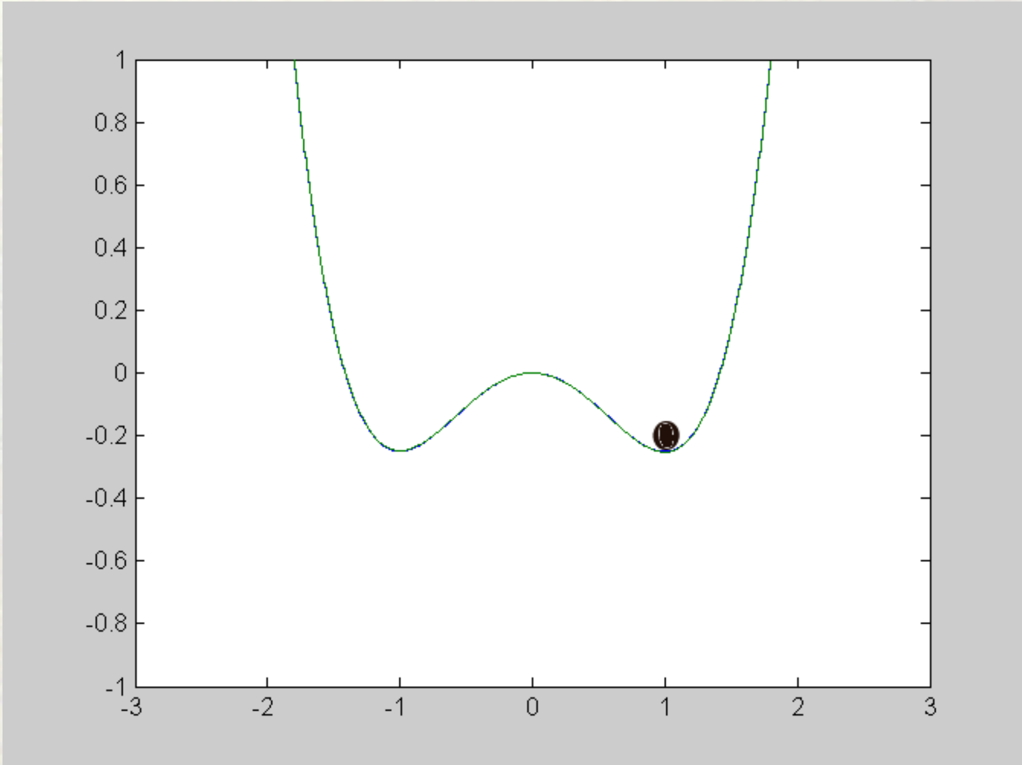
Marble in a cartoon: let's play dynamics!

$$\ddot{x} = -V'(x) - \gamma\dot{x} + A(t)$$

$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \qquad A(t) = A\sin(\omega t)$$



The **Stochastic Resonance** phenomenon

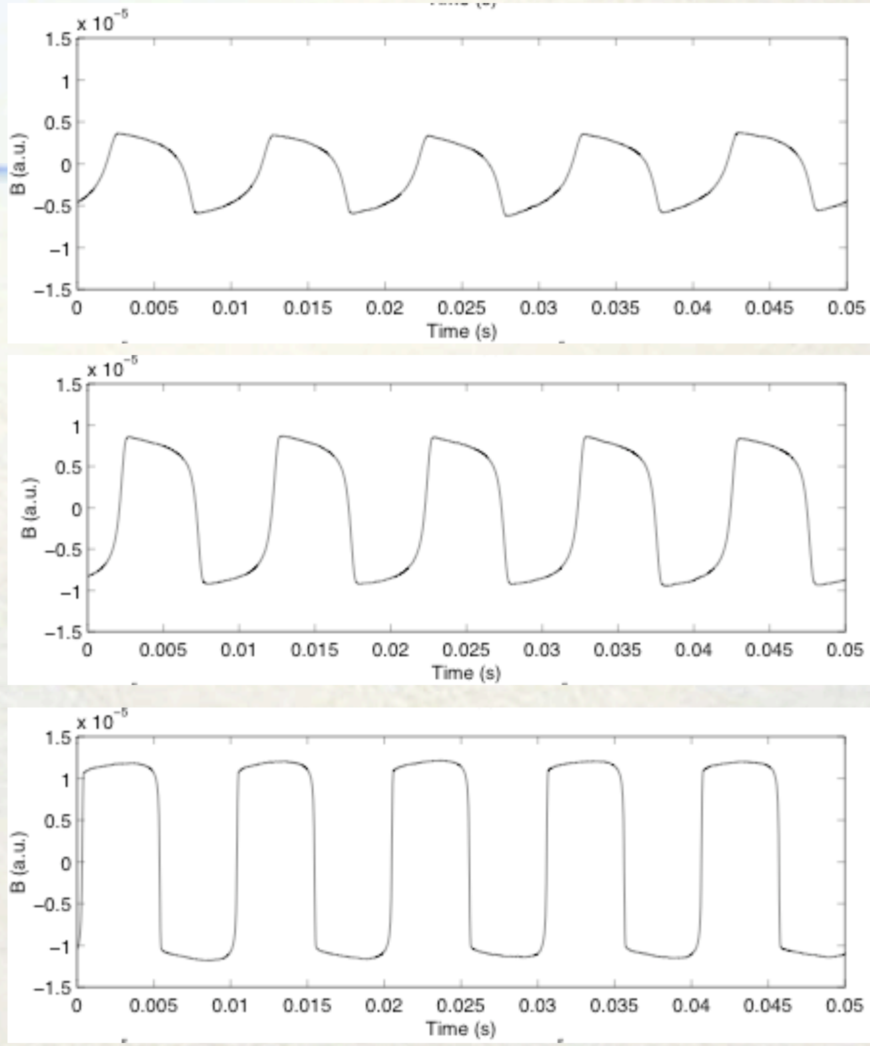
Let's consider our very basic ingredients:

$$\ddot{x} = -V'(x) - \gamma\dot{x} + A(t)$$

$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \qquad A(t) = A\sin(\omega t)$$

Let's look for the time evolution of x(t)

How does it look?



A = 3

A= 4

A= 5

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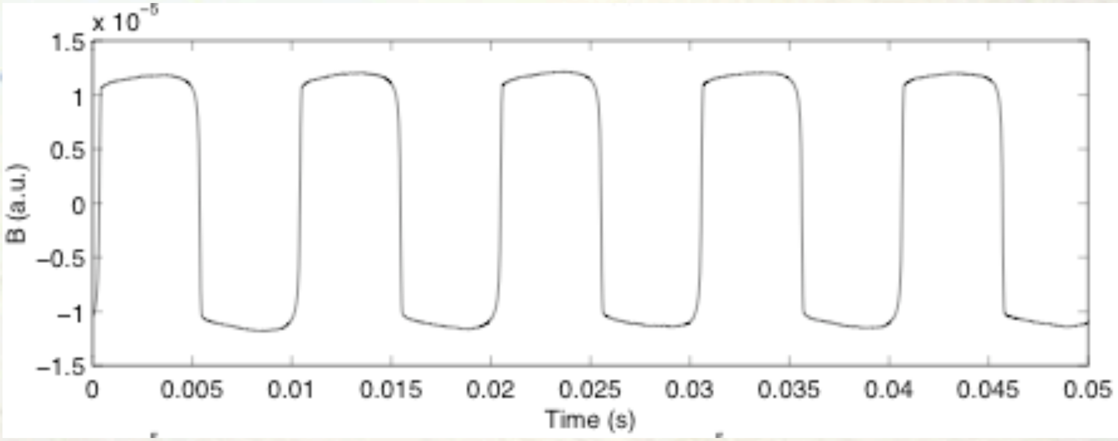
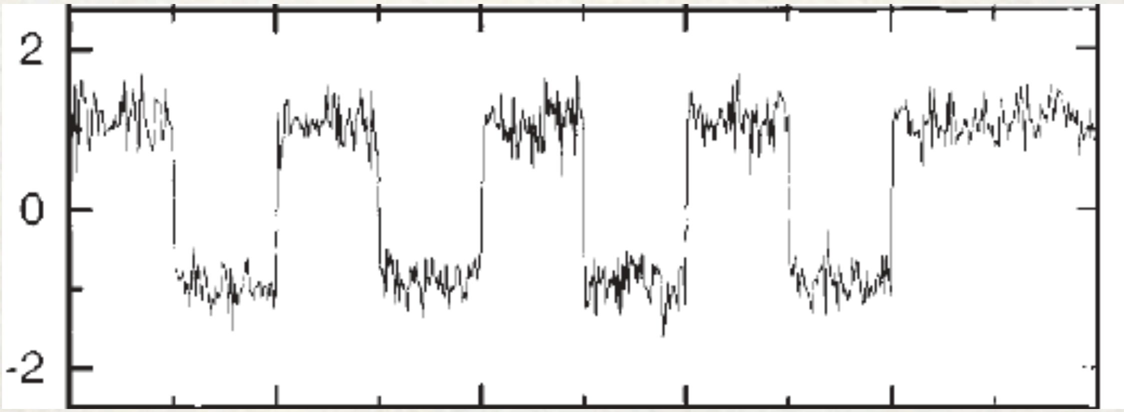
What if we add the noisy force?



No noise

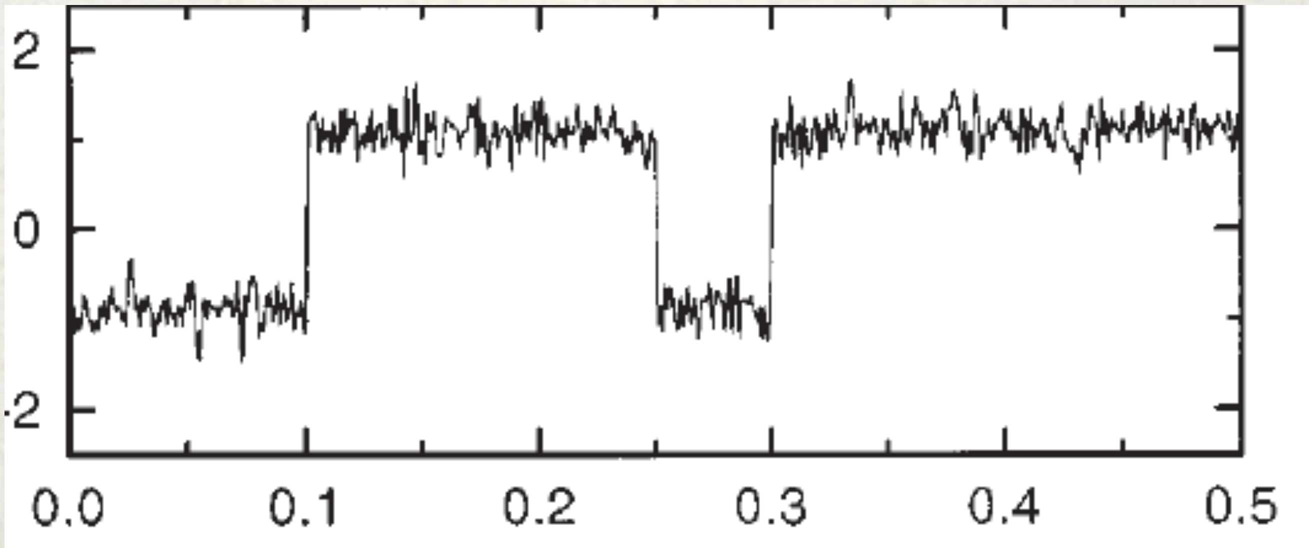


With noise

What if the periodic driving is to small
to let the marble to jump??

The marble would stay trapped into one well unless
The noise provide some help…

For **small** noise we will have **occasional** jumps



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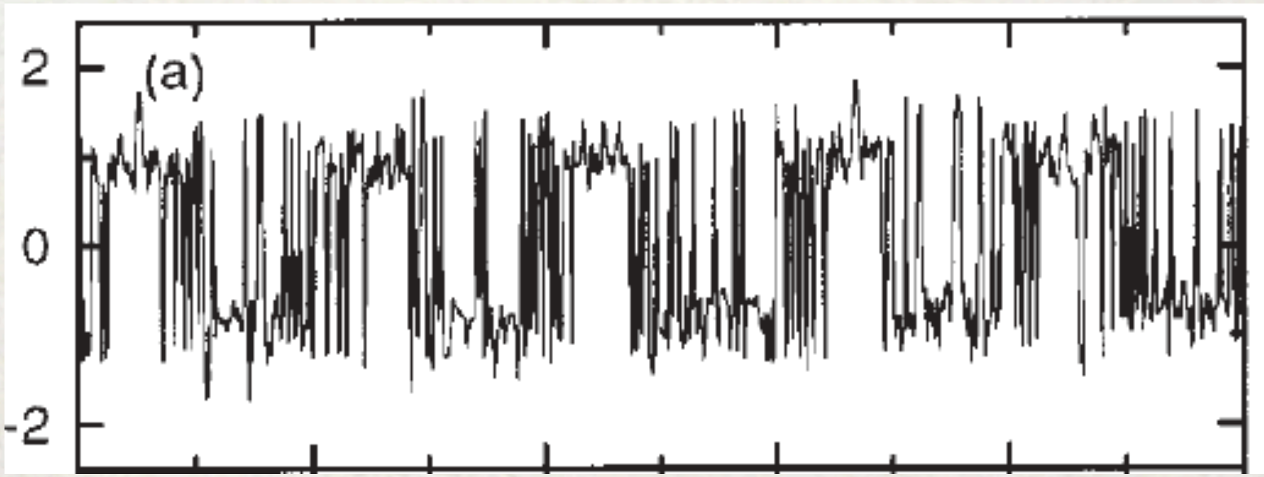
What if the periodic driving is to small
to let the marble to jump??

The marble would stay trapped into one well unless
The noise provide some help…

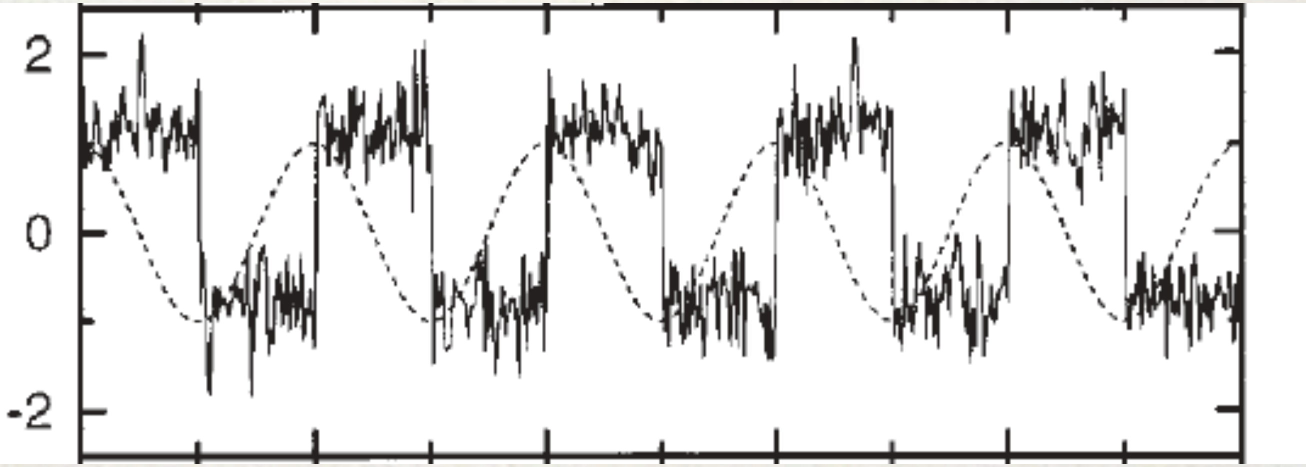
For **large** noise we will have **frequent** jumps



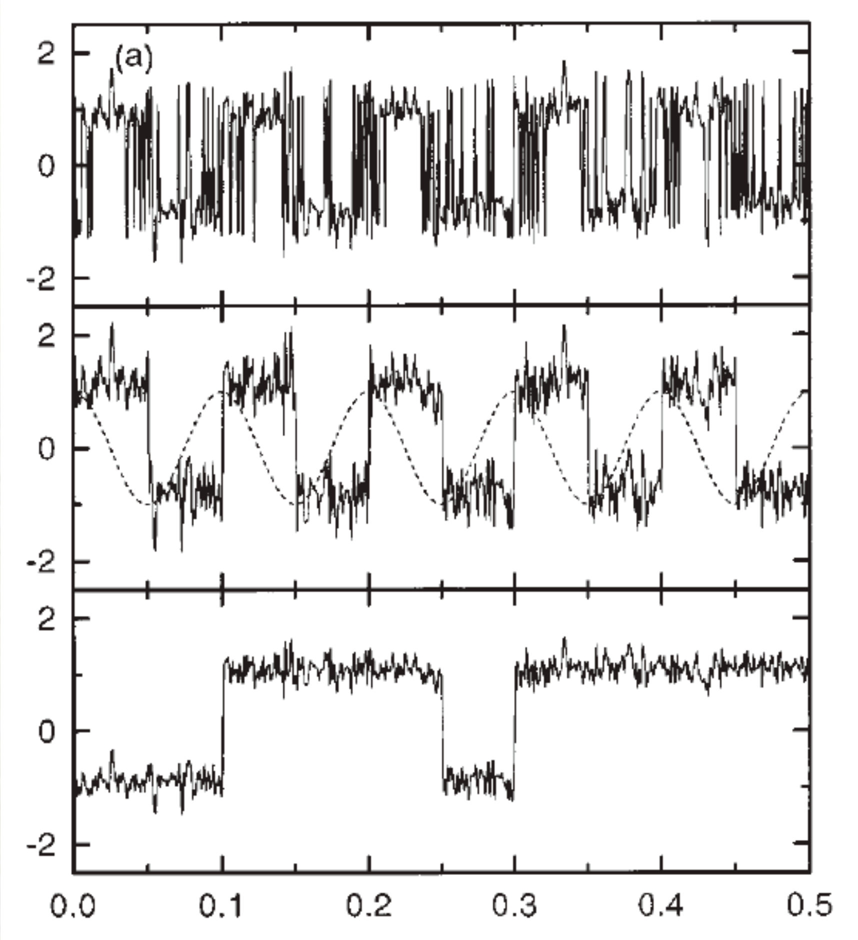
What if the periodic driving is to small
to let the marble to jump??

And in between ?

It exists an optimal noise intensity that produces the Jumps in synchrony with the forcing !!!



The **Stochastic Resonance** Phenomenon



Large noise

Optimal noise

Small noise

The **Stochastic Resonance** Phenomenon

First paper in 1981 by Benzi, Parisi, Sutera, Vulpiani.

Since then more than 3000 papers (to date)…

Stochastic Resonance

1998SR2008

Perugia, Aug. 17-21, 2008

THE CONFERENCE

The European Physical Journal B

Vol. 69 No. 1 (May I 2009)

Special Issue: Stochastic Resonance

Other "surprising" noise induced phenomena

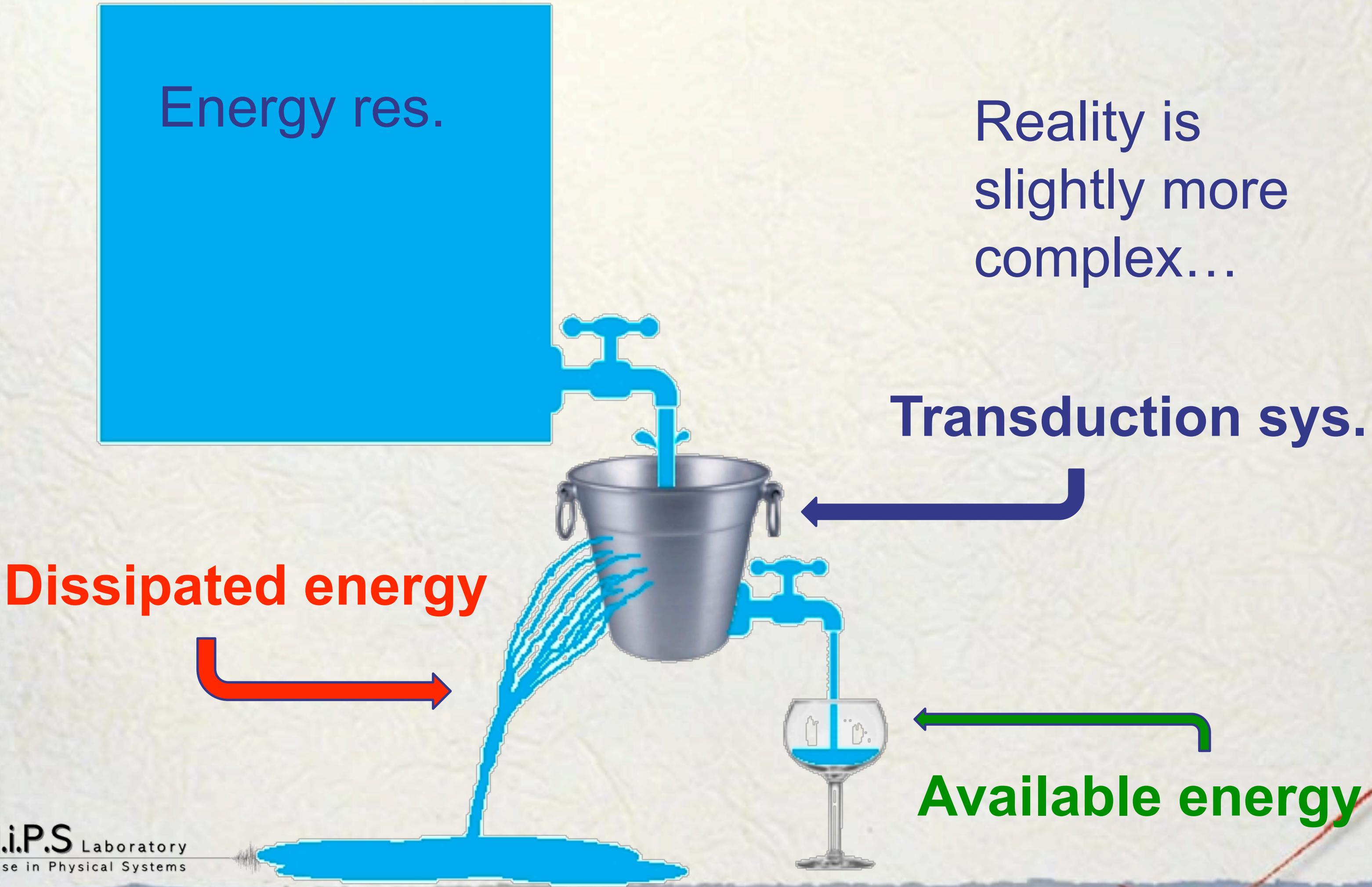
1. Resonant Trapping
2. Dithering Effect
3. Resonant Activation
4. Brownian Ratchet
5. Resonant crossing
6. …
7. **Energy harvesting (noise harvesting)**

Energy harvesting basic ideas

ENERGY

Unlimited source of free energy, readily available for multiple uses…

Energy harvesting basic ideas



Reality is slightly more complex…

Energy harvesting basic ideas

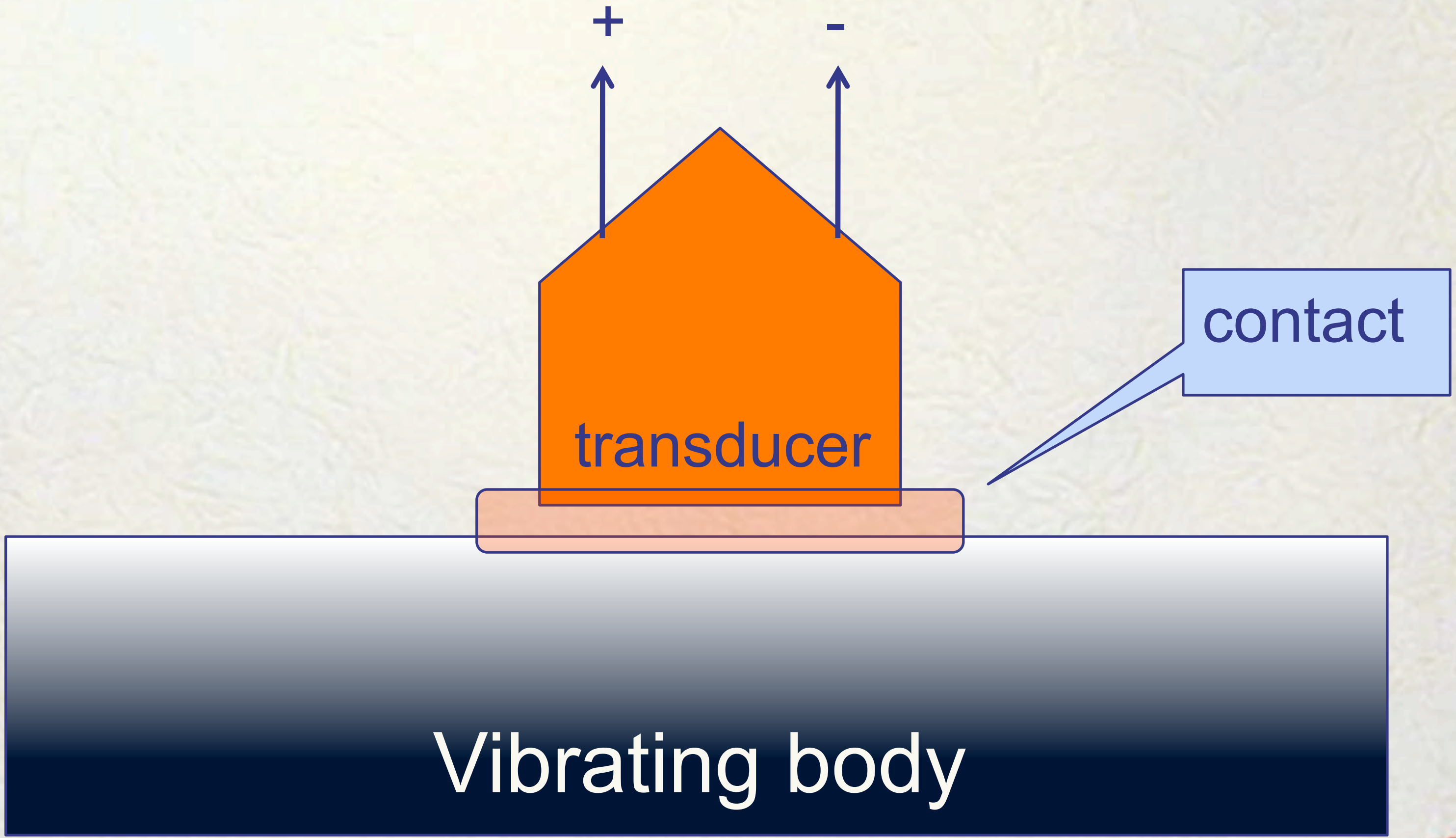
Kinetic energy

- wind
- sound
- Falling bodies
- vibrations
- water waves and tides

Focus on vibrations of solid bodies….

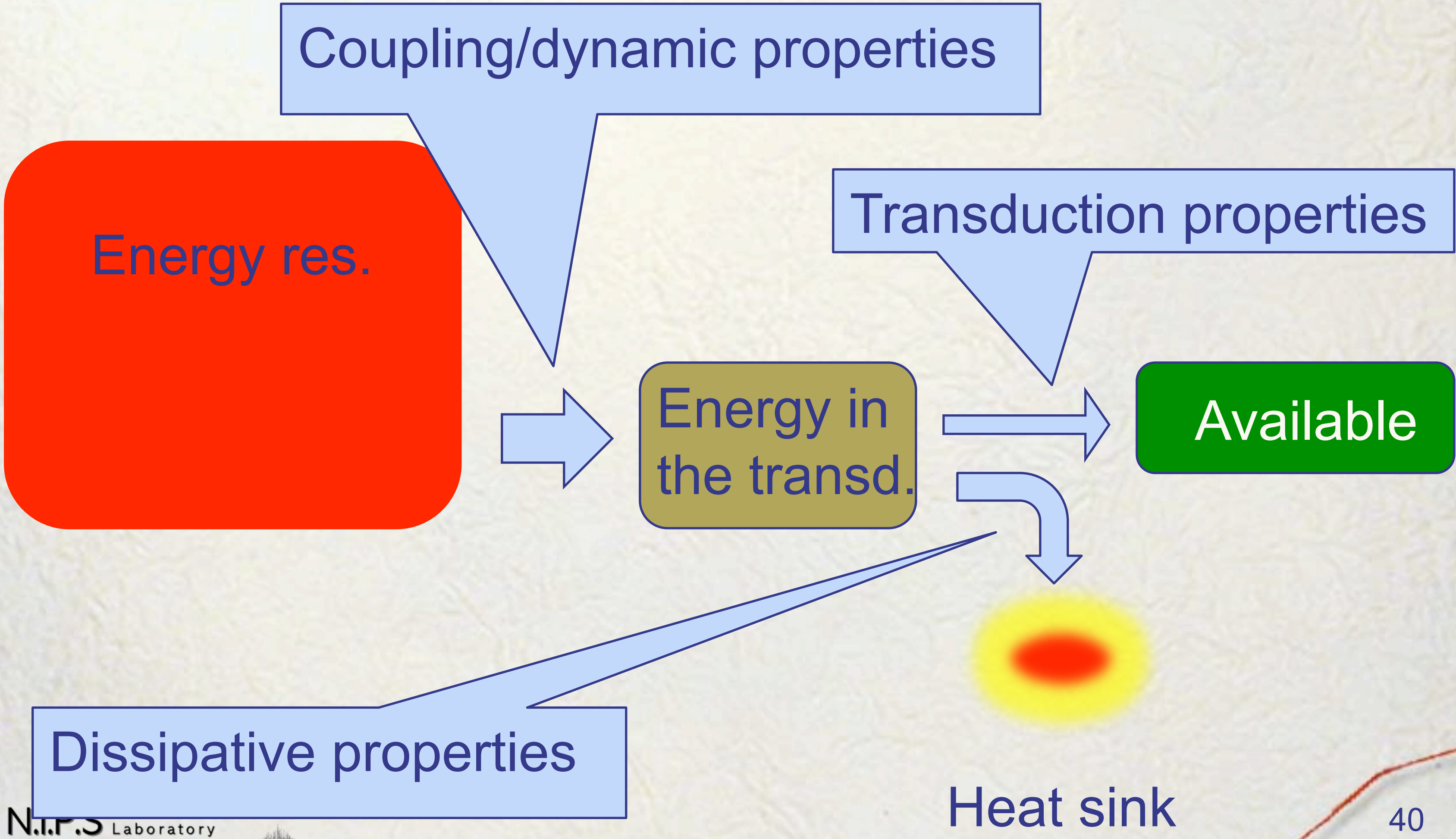
Vibrations energy harvesting

Basic Scheme



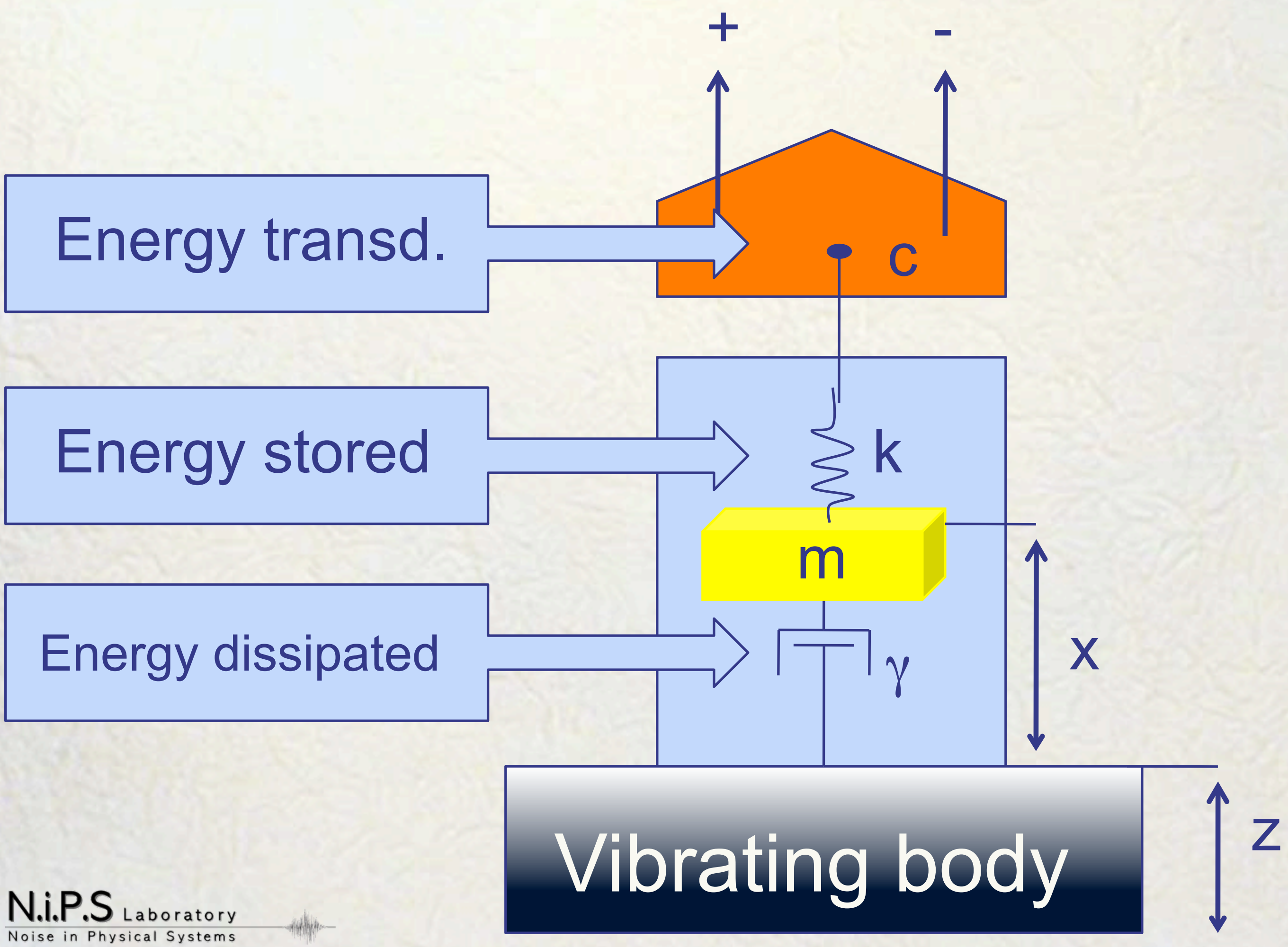
Vibrations energy harvesting

Energy budget

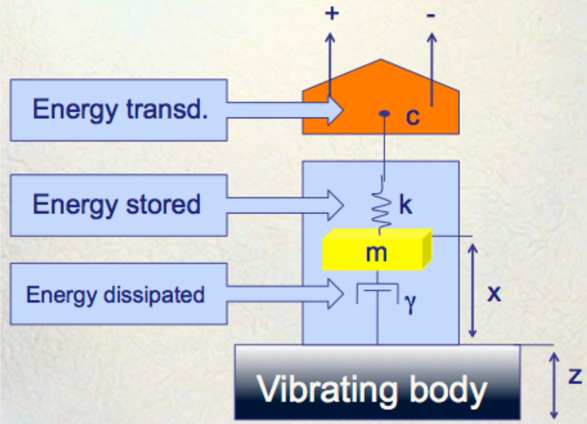


Vibrations energy harvesting

Dynamical model



Vibrations energy harvesting



Dynamical model

$$m\ddot{x} = -\frac{dU(x)}{dx} - \gamma\dot{x} - c(x,V) + \zeta_z$$

Where:

	$U(x)$	Represents the Energy stored
	$\gamma\dot{x}$	Accounts for the Energy dissipated
	$c(x,V)$	Accounts for the Energy transduced
	ζ_z	Accounts for the input Energy

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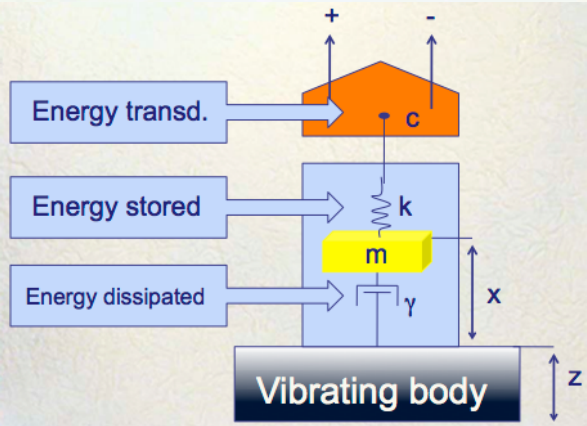
Vibrations energy harvesting



Dynamical model

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - c(x,V) + \zeta_z \\ \dot{V} = F(\dot{x},V) \end{cases}$$

Equations that link the vibration-induced displacement with the Voltage

Vibrations energy harvesting

Dynamical model

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - c(x,V) + \zeta_z \\ \dot{V} = F(\dot{x},V) \end{cases}$$

Details depend on the physics…

Three main transduction mechanisms…

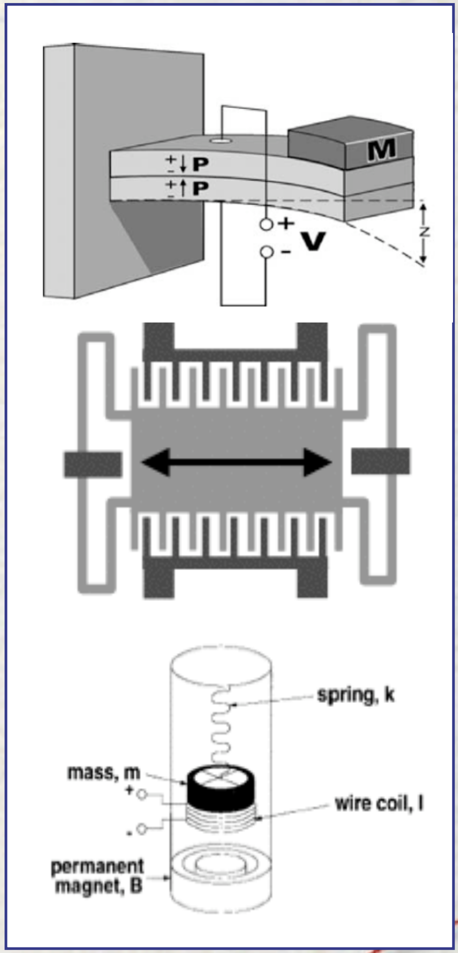
Vibrations energy harvesting

Transduction mechanisms

1. Piezoelectric: dynamical strain is converted into voltage difference.

2. Capacitive: geometrical variations induce voltage difference

3. Inductive: dynamical oscillations of magnets induce electric current in coils



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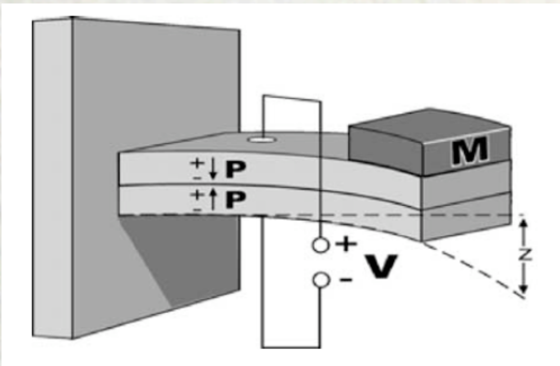
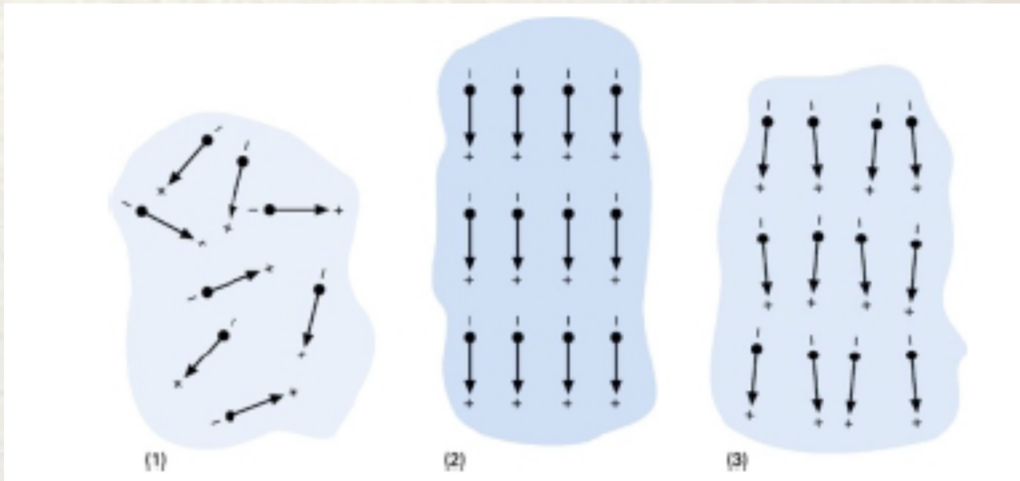
Transduction mechanisms

1 Piezoelectric: dynamical strain is converted into voltage difference.





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Transduction mechanisms

1 Piezoelectric: dynamical strain is converted into voltage difference.

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - K_V V + \zeta_z \\ \dot{V} = K_c\dot{x} - \dfrac{1}{\tau_p}V \end{cases}$$

The Physics of piezo materials

The available power is proportional to V^2

Vibrations energy harvesting

Transduction mechanisms

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - K_V V + \zeta_z \\ \dot{V} = K_c \dot{x} - \dfrac{1}{\tau_p} V \end{cases}$$

$U(x)$ Represents the Energy stored

When $U(x) = \frac{1}{2}kx^2$ it is called a linear system

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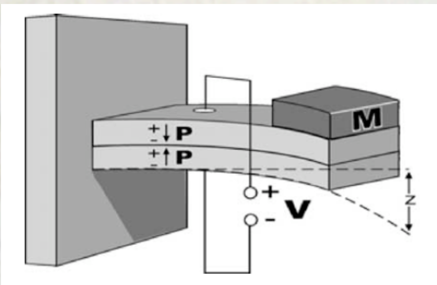
Vibrations energy harvesting

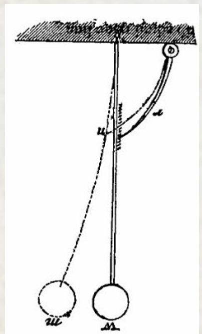
Linear systems

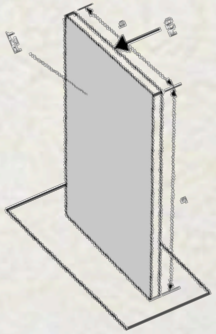
When $U(x) = \frac{1}{2} k x^2$ it is called a linear system

Linear systems have some interesting features… (and engeneers like them most)

1) There exist a simple math theory to solve the eq.s
2) They have a resonant behaviour (resonance freq.)
3) They can be “easily” realized with catilevers and pendula

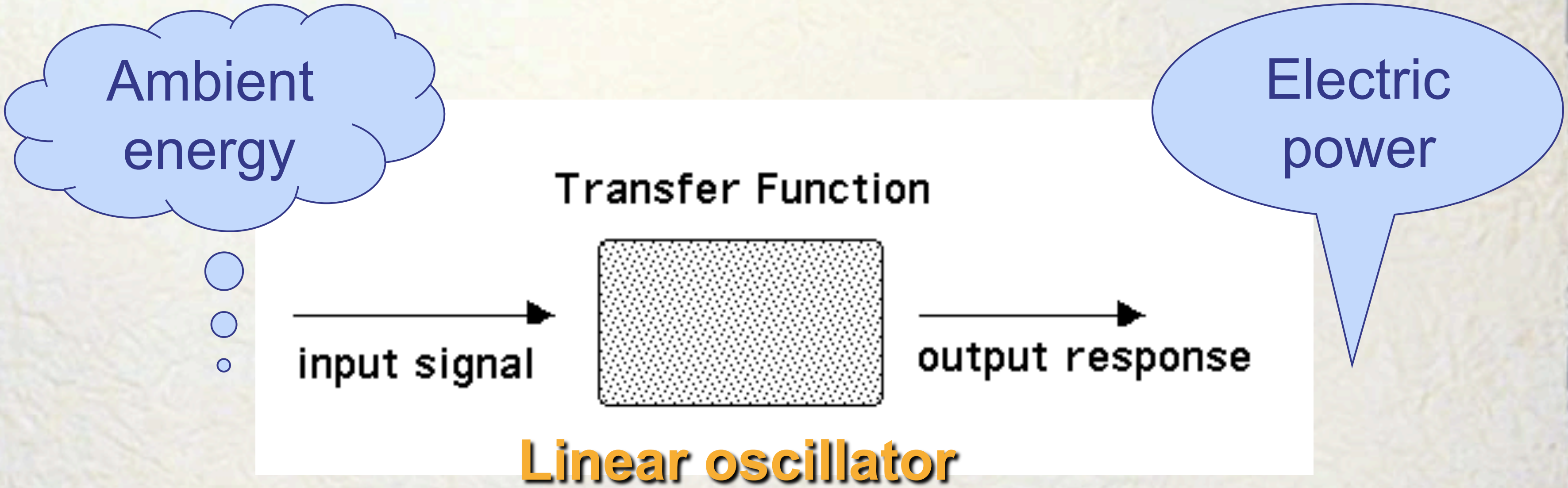






Vibrations energy harvesting

Linear systems

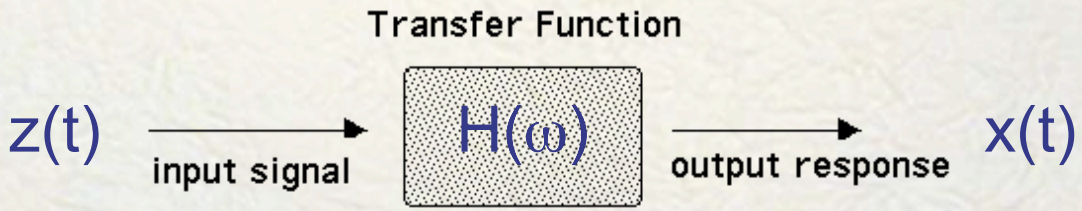


The transfer function is a math function of the frequency, in the complex domain, that can be used to represent the performance of a linear system

Vibrations energy harvesting

Linear systems

In a linear system, thanks to the transfer function $H(\omega)$, the output spectrum can be obtained from the input spectrum through a simple multiplication…



$$S_x(\omega) = |H(\omega)|^2 \, S_z(\omega)$$

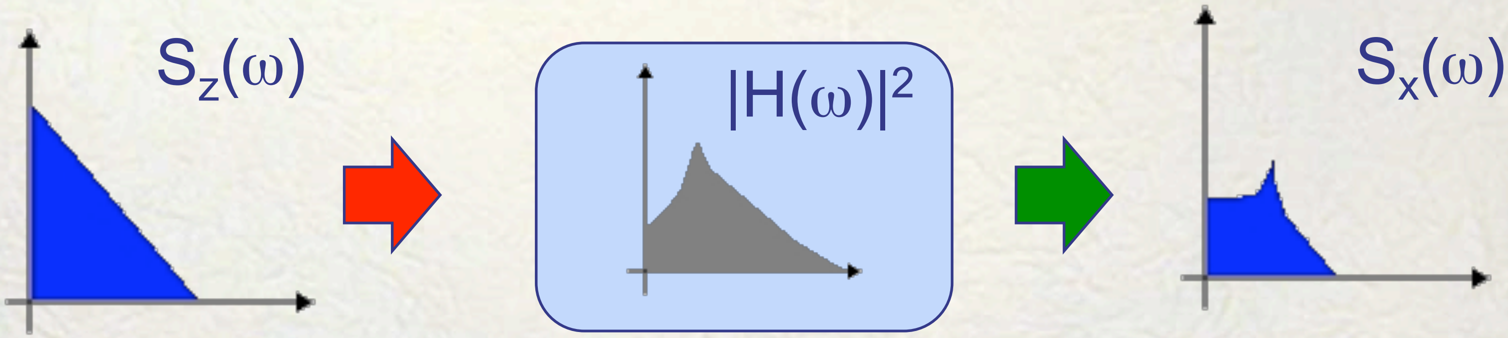
Output power spectrum

Input power spectrum

Vibrations energy harvesting

Linear systems

The transfer function is important because it acts as a filter on the incoming energy…



Freq. spectrum of the available energy

Transfer function of the transducer

Freq. spectrum of the usable energy

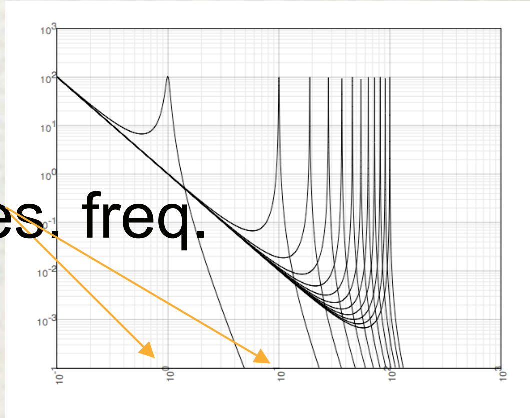
$$S_x(\omega) = |H(\omega)|^2 \, S_z(\omega)$$

Vibrations energy harvesting

Linear systems

For a linear system the transfer function presents one or more peeks corresponding to the resonace frequencies and thus it is efficient mainly when the incoming energy is abundant in that regions…

This is a serious limitation when you want to build a small energy harvesting system…



Why ?!!

Vibrations energy harvesting

Linear systems

For two main reasons…

(1) the frequency spectrum of available vibrations instead of being sharply peaked at some frequency is usually very broad.

(2) The frequency spectrum of available vibrations is particularly rich in energy in the low frequency part… and it is very difficult, if not impossible, to build small low-frequency resonant systems…

Let's see some examples…

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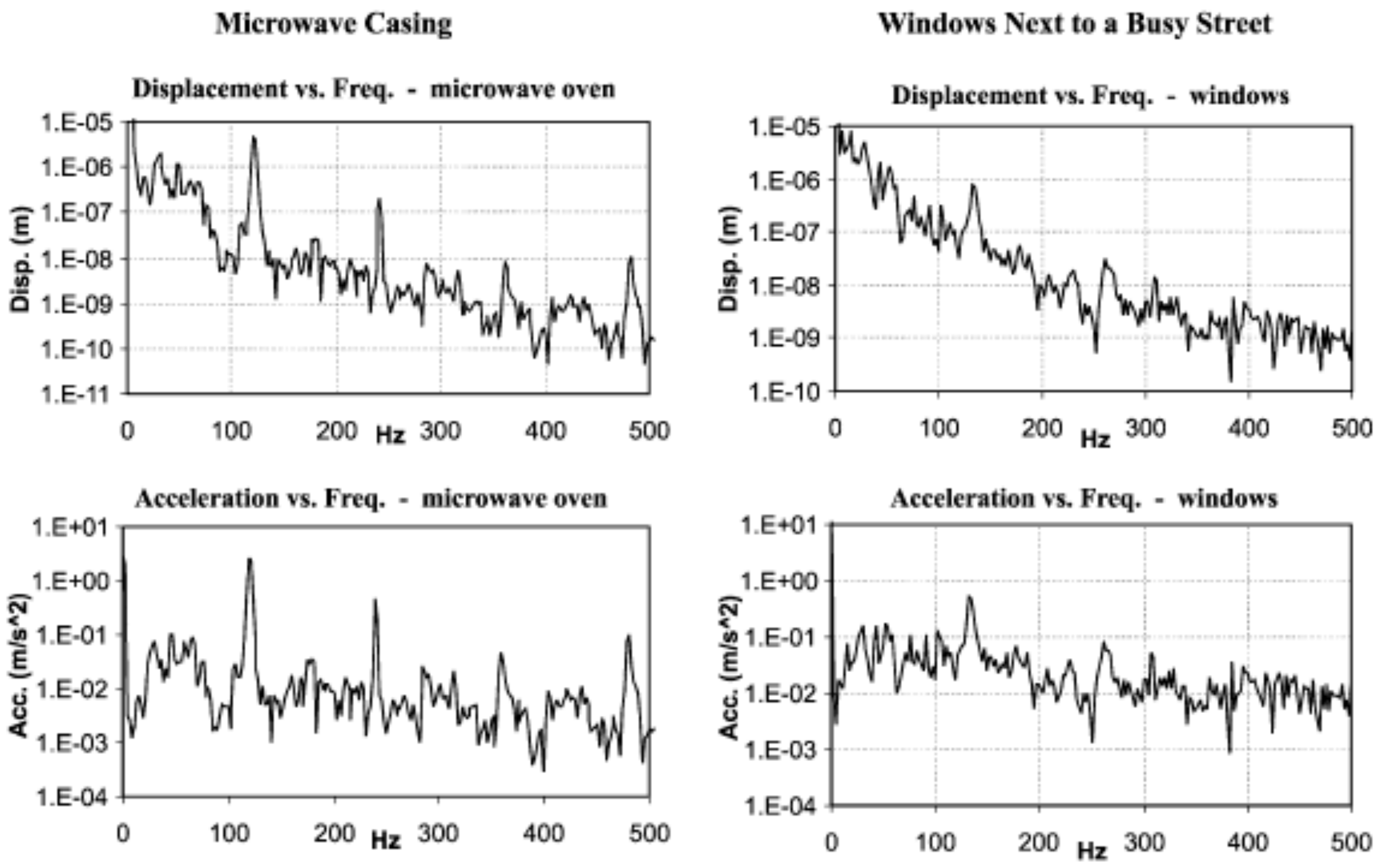
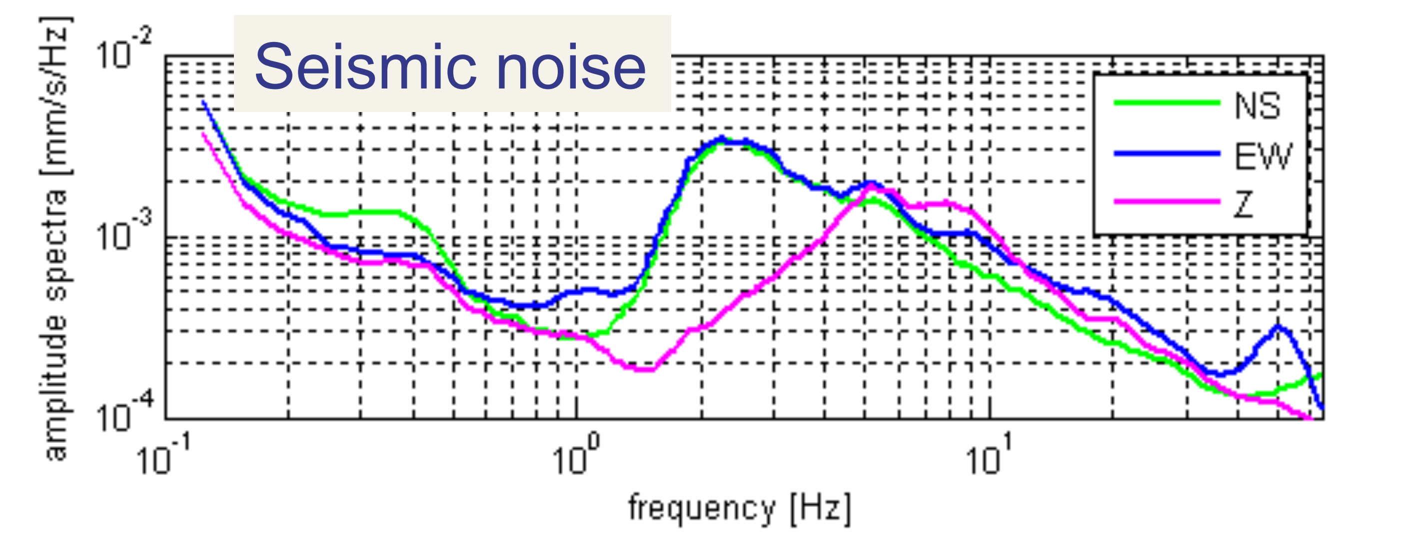
Vibrations energy harvesting

S. Roundy et al. / Computer Communications 26 (2003) 1131–1144

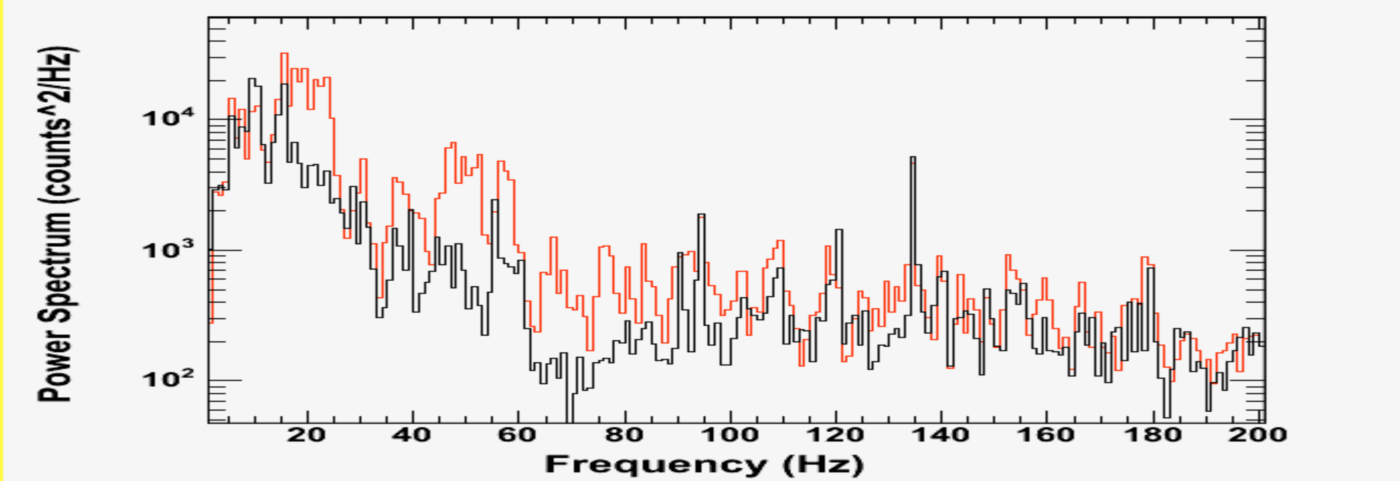


Vibrations energy harvesting

Seismic noise



Acoustic noise – quiete working env.



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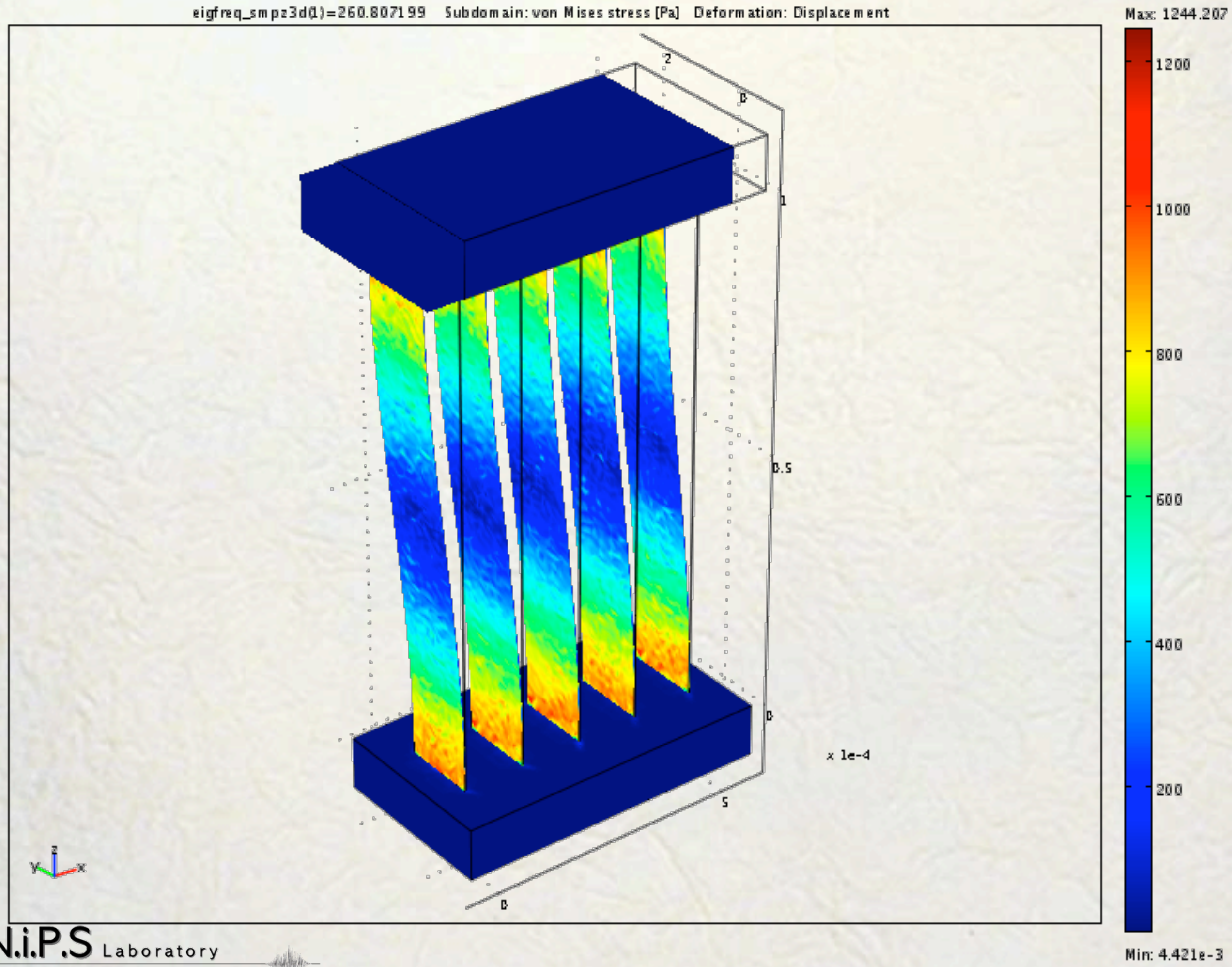
Vibrations energy harvesting

Micro energy harvesting system…



25 μm thick
1 mm high

Freq. 10 KHz

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Vibrations energy harvesting

Whish list for the perfect vibration harvester

1) Capable of harvesting energy on a broad-band
2) No need for frequency tuning
3) Capable of harvesting energy at low frequency

↓

1) Non-resonant system
2) “Transfer function” with wide frequency resp.
3) Low frequency operated

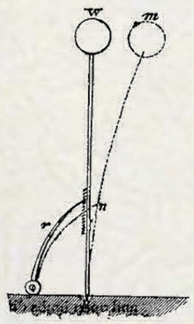
Vibrations energy harvesting

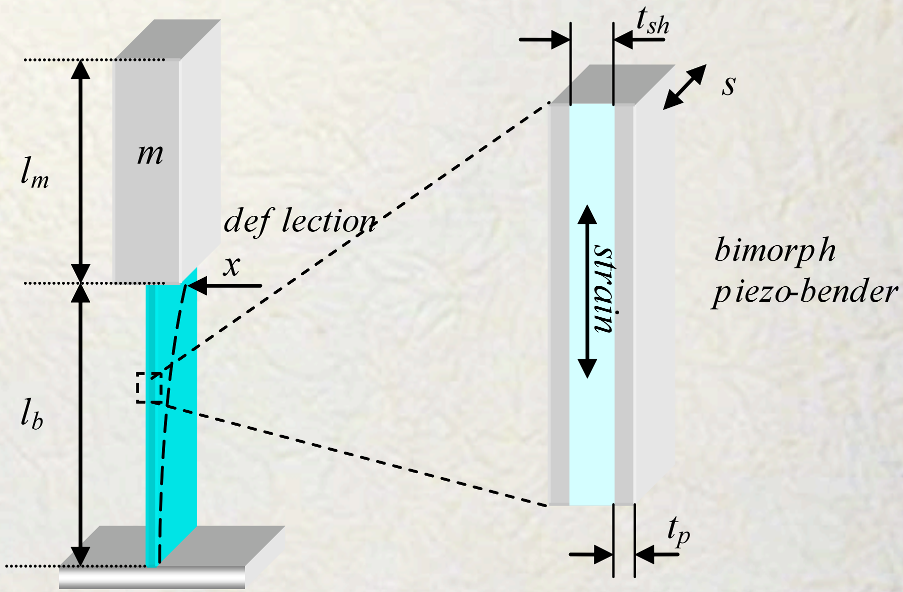
NON-Linear mechanical oscillators

1) Non-resonant system
2) “Transfer function” with wide frequency resp.
3) Low frequency operated

Example…

Inverted pendulum





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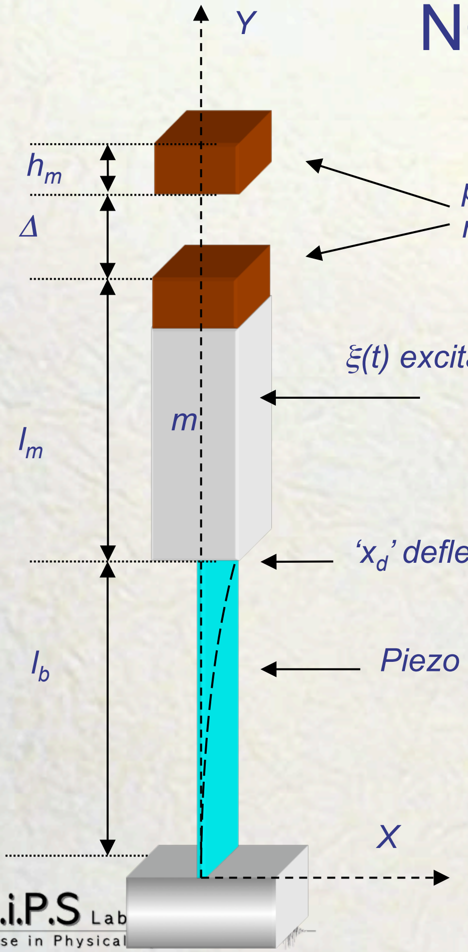
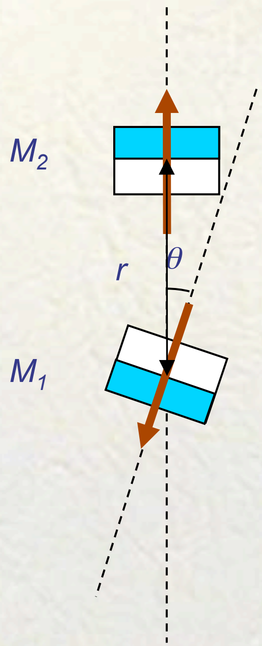
F. Cottone, PhD Thesys, Perugia 2007

Vibrations energy harvesting

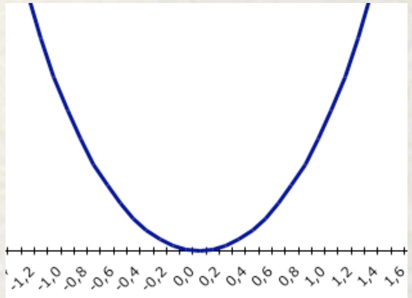
NON-Linear mechanical oscillators

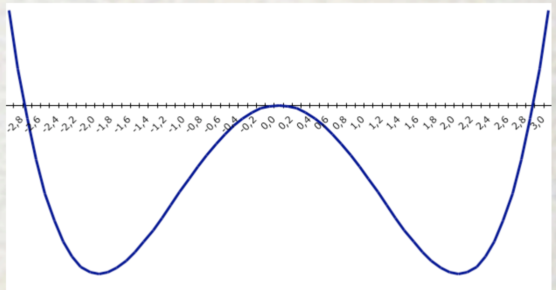
NON-Linear Inverted pendulum





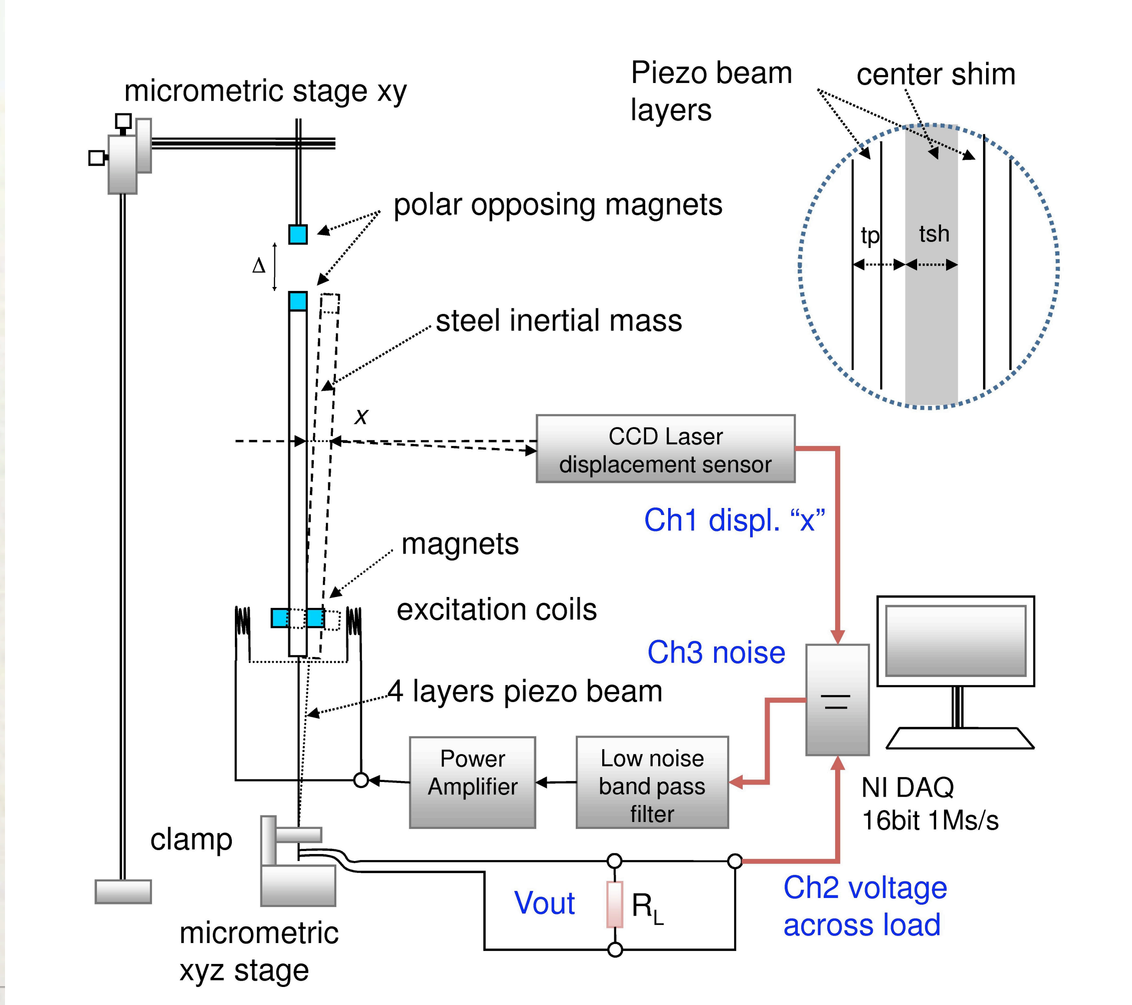
b)





Vibrations energy harvesting

NON-Linear mechanical oscillators



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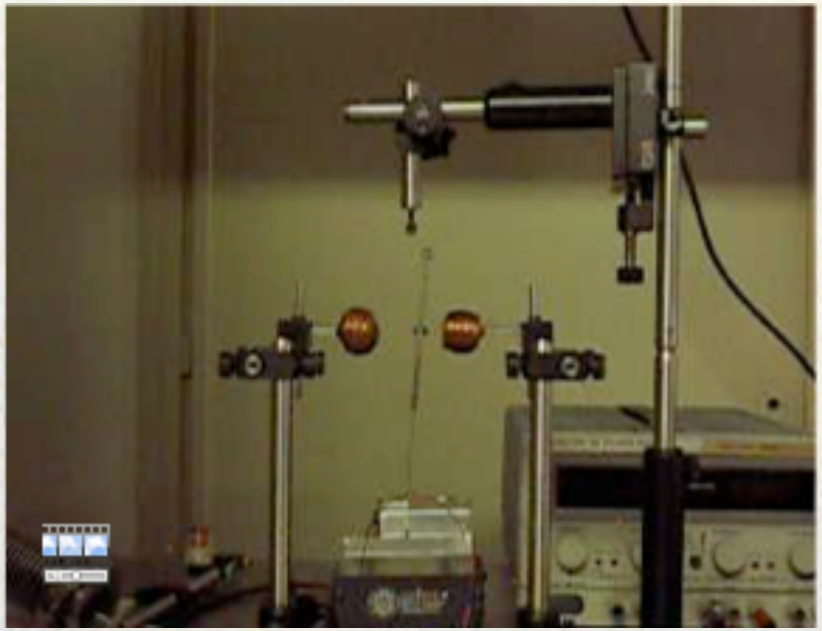
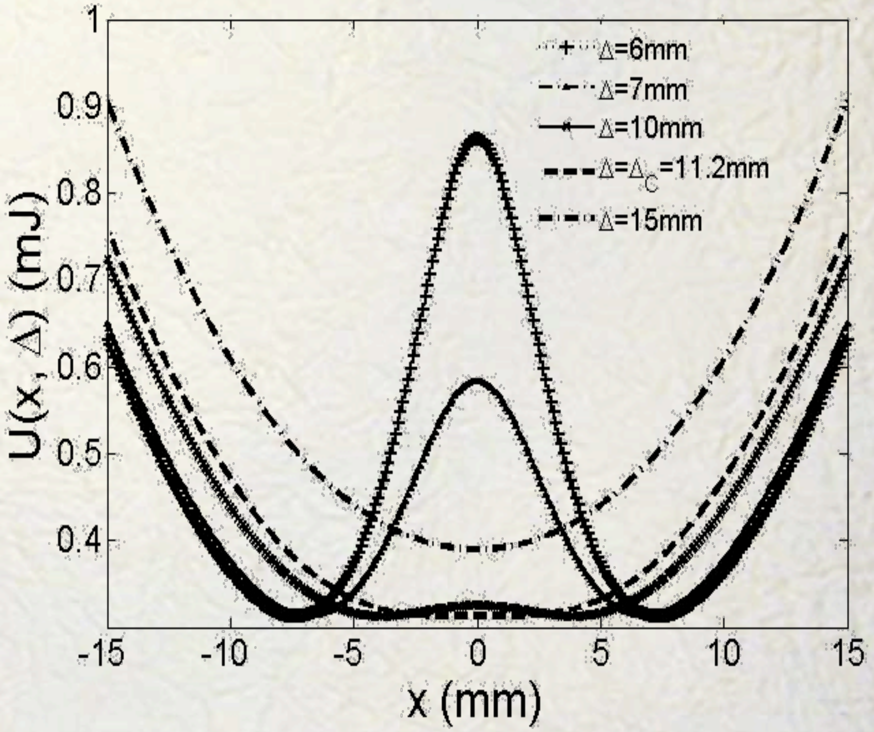
Vibrations energy harvesting

NON-Linear mechanical oscillators





http://www.nipslab.org/node/1676

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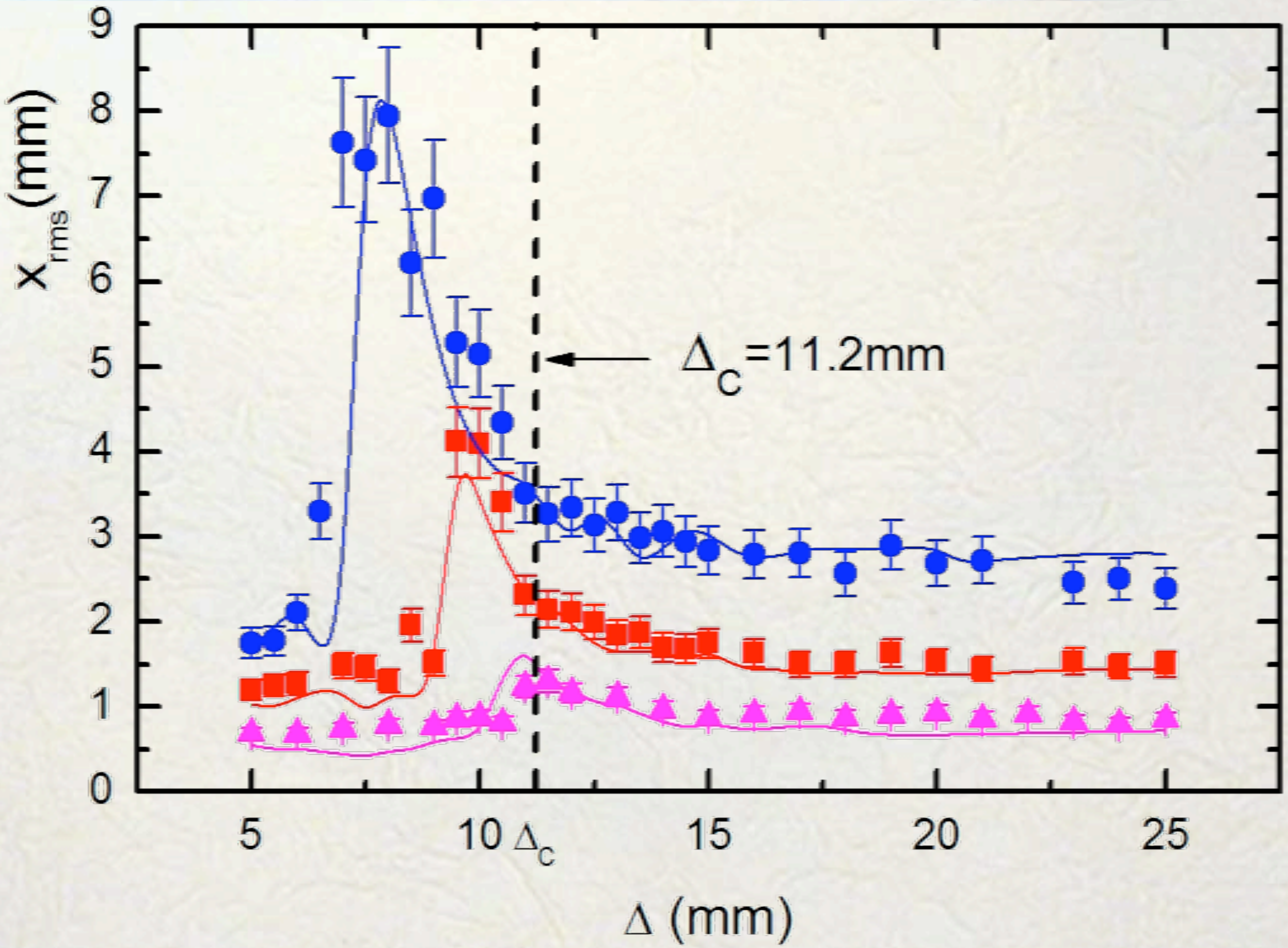
Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni , Physical Review Letters, 102, 080601 (2009)

Vibrations energy harvesting

NON-Linear mechanical oscillators



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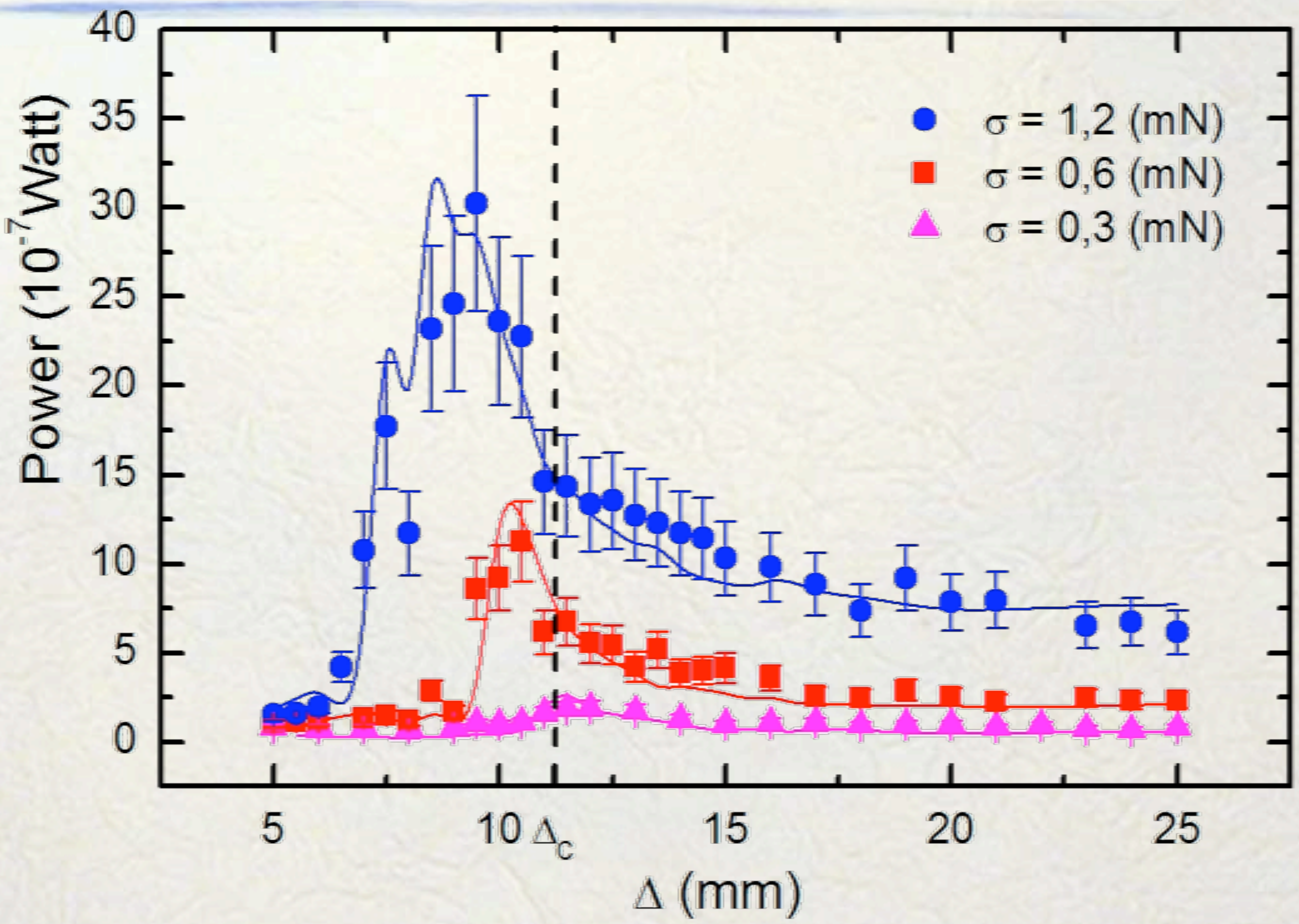

Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni , Physical Review Letters, 102, 080601 (2009)

To think about…

1) Non resonant (i.e. non-linear) mechanical oscillators can outperform resonant (i.e. linear) ones*

2) Non-linear systems are more difficult to treat

3) Bistability is not the only nonlinearity available… see:
L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)