ENERGY HARVESTING
TRANSUDCERS - ELECTROSTATIC
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Three Types of Electromechanical Lossless Transduction

1. **Electrodynamic** (also called **electromagnetic** or inductive): motor/generator action is produced by the current in, or the motion of an electric conductor located in a fixed transverse magnetic field (e.g. voice coil speaker)

2. **Piezoelectric**: motor/generator action is produced by the direct and converse piezoelectric effect – dielectric polarization gives rise to elastic strain and vice versa (e.g. tweeter speaker)

3. **Electrostatic**: motor/generator action is produced by variations of the mechanical stress by maintaining a potential difference between two or more electrodes, one of which moves (e.g. condenser microphone)

Credit: This classification and much of the flow from Electromagnetic section is based on the 2013 PowerMEMS presentation by Prof. David Arnold at the University of Florida
Outline for Short Course

• Introduction and Linear Energy Harvesting
• **Energy Harvesting Transducers**
  – Electromagnetic
  – Piezoelectric
  – Electrostatic
• Wideband and Nonlinear Energy Harvesting
• Applications
FUNDAMENTALS OF ELECTROSTATIC TRANSDUCTION
Basic Capacitor Relationship

For a parallel plate capacitor

\[ C = \frac{\epsilon A}{d} \]

\[ U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} \]

where \( \epsilon \) is the permittivity between the plates (usually air \( \epsilon_0 = 8.85 \times 10^{-12} \)), \( V \) is the voltage across the plates, and \( Q \) is the charge stored on the plates (\( Q = CV \)).

If capacitance changes due to external excitation, the energy stored in the capacitor will change, and this energy can be harvested.
Three Types of Electrostatic Harvesters

Out of plane

Overlapping area change

Permittivity change
Energy Conversion Cycles
Voltage Constrained

(1) – (2) Capacitor charged at $C_{\text{max}}$ to $V_{\text{max}}$.

(2) – (3) Voltage held at $V_{\text{max}}$ while capacitance reduces to $C_{\text{min}}$. Charge is either returned to voltage source or to an external circuit.

(3) – (1) Remaining charge recovered at capacitance of $C_{\text{min}}$.

$$E_{1231} = \frac{1}{2} (C_{\text{max}} - C_{\text{min}}) V_{\text{max}}^2$$
Energy Conversion Cycles  
Charge Constrained

(1) – (2’) Capacitor charged at $C_{\text{max}}$ to $V_{\text{in}}$.

(2’) – (3) Capacitor is open circuit (constant charge) while capacitance reduces to $C_{\text{min}}$. In order for charge to remain the same, voltage on capacitor increases to $V_{\text{max}}$.

(3) – (1) Charge is recovered at much higher voltage ($V_{\text{max}}$) and therefore higher energy than it was injected.

\[
E_{12,31} = \frac{1}{2} (C_{\text{max}} - C_{\text{min}}) V_{\text{max}} V_{\text{in}}
\]

\[
E_{1231} = \frac{1}{2} \left( \frac{1}{C_{\text{min}}} - \frac{1}{C_{\text{max}}} \right) Q_0^2
\]
Some Variable Capacitance Structures

Interdigitated Comb Fingers
Gap Closing

Interdigitated Comb Fingers
Changing Overlap

Motion
Some Variable Capacitance Structures

In plane, overlapping area harvester with patterned electrodes

![Diagram of a harvester with patterned electrodes in motion]
Some Intermediate Comments

- Natural for implementation in silicon MEMS where variable capacitance structures are common
  - But, proof mass is extremely low, so power is usually very low
- Energy densities are usually low due to the low permittivity of air unless the voltages are very high
- Needs some sort of rechargeable battery to prime the capacitor, or quite complicated control circuitry
- So, recently, electret based harvesters have become much more common
Electrets

- “Electret is a dielectric material that has a quasi-permanent electric charge or dipole polarisation. An electret generates internal and external electric fields, and is the electrostatic equivalent of a permanent magnet.” - Wikipedia

- Electrets are characterized by their surface charge density and stability over time

- SiO$_2$ forms an electret with very high surface charge density, but its stability over time is a concern

- Perfluorinated polymers provide better stability

- CYTOP (Asahi Glass Co.) has become a highly used electret material for this application
Electrets

Figure 9. Standard electrets for electret-based electrostatic converters (a) dipole orientation and (b) charge injection

Figure 10. Corona discharge device (a) principle and (b) photo (CEA-LETI)

## Example Electret Properties

<table>
<thead>
<tr>
<th>Electret</th>
<th>Max thickness (μm)</th>
<th>Relative permittivity</th>
<th>Charge density [mC/m²]</th>
<th>Surface voltage @ max thick</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teflon</td>
<td>100</td>
<td>2.1</td>
<td>0.1 – 0.25</td>
<td>~ 600</td>
</tr>
<tr>
<td>SiO₂</td>
<td>3</td>
<td>4</td>
<td>5 – 10</td>
<td>~ 500</td>
</tr>
<tr>
<td>Parylene</td>
<td>10</td>
<td>3</td>
<td>0.5 - 1</td>
<td>~ 200</td>
</tr>
<tr>
<td>CYTOP</td>
<td>20</td>
<td>2</td>
<td>1 - 2</td>
<td>~ 1000</td>
</tr>
</tbody>
</table>

Electret Harvester – Operating Principles

Figure 13. Electret-based electrostatic conversion – Concept

Figure 14. Electret-based electrostatic conversion – Charge circulation

Electret Equivalent Circuit

Figure 15. Electrical equivalent model of electret-based electrostatic converters

Electret Harvester Theory

The governing equation for this circuit is:

$$\frac{dQ_2}{dt} = \frac{1}{1 + \frac{C_p}{C(t)}} \left[ V_s - Q_2 \left( \frac{1}{RC(t)} - \frac{C_p}{C(t)^2} \frac{dC(t)}{dt} \right) \right]$$

where $C_p = C_{par}$ is the parasitic capacitance.

This can be re-written in terms of load voltage, $V$. (Note, this written as $U$ in the figure, which is confusing so I use $V$.)

$$\frac{dV}{dt} = -\frac{1}{C_p + C(t)} \left[ V \left( \frac{1}{R} + \frac{dC(t)}{dt} \right) - \frac{dC(t)}{dt} V_s \right]$$

The power is then just:

$$P_{rms} = \frac{1}{2} \frac{|V|^2}{R}$$

Note: electrets are sometimes characterized by surface voltage ($V_s$) as in the equations above, and sometimes by charge density, $\sigma$, where

$$V_s = \frac{\sigma d}{\epsilon \epsilon_0}.$$
Electret Harvester Mechanics

Electrostatic force will act to minimize energy in the system, or maximize capacitance.

\[ F_e = -\frac{d}{dz} (W_{elec}) = -\frac{d}{dz} \left( \frac{1}{2} C(z)(V(z) - V_s)^2 \right) \]

where \( W_{elec} \) = electrostatic force, \( V(z) \) = load voltage as a function of position
Electret Harvester Mechanics

Remember from Linear VEH theory

\[ m \ddot{z} + (b_m + b_e) \dot{z} + kz = -m \ddot{y} \]

where \( b_e \dot{z} \) is the electrically induced force. Substituting:

\[ m \ddot{z} + b_m \dot{z} + F_e + kz = -m \ddot{y} \]

And the governing equations for the system are:

\[
\begin{align*}
m \ddot{z} + b_m \dot{z} & - \frac{d}{dz} \left( \frac{1}{2} C (V - V_s)^2 \right) + kz = -m \ddot{y} \\
\dot{V} & = -\frac{1}{C_p + C} \left[ V \left( \frac{1}{R + \frac{dC}{dt}} \right) - \frac{dC}{dt} V_s \right]
\end{align*}
\]

This must be solved numerically as a simple expression for \( C(z) \) is often not available.
Capacitance Expressions

Example 1, gap closing oscillator:

\[ C(z) = \frac{\varepsilon_0 A}{z + g + \frac{d}{\varepsilon}} \]

where \( A \) is the total maximum electrode overlap area, \( g \) is the thickness of the air gap, and \( d \) is the thickness of the electret, \( z \) is the displacement of counter electrode.

\[ \frac{dC(z)}{dz} = \frac{-\varepsilon_0 A}{\left(z + g + \frac{d}{\varepsilon}\right)^2} \]

\[ F_e = \frac{1}{2} \frac{\varepsilon_0 A}{\left(z + g + \frac{d}{\varepsilon}\right)^2} (V - V_s)^2 \]
Capacitance Expressions

Example 2, overlapping area patterned electrodes:

The maximum capacitance is:

\[ C_{\text{max}} = \frac{\varepsilon_0 A}{g + \frac{d}{\varepsilon}} \]

where \( A \) is the total maximum electrode overlap area, \( g \) is the thickness of the air gap, and \( d \) is the thickness of the electret.

See Boisseau et. al. 2012 for a discussion of minimum capacitance for the common overlapping area with patterned electrode architecture.

\[ C(z) = \frac{C_{\text{max}} + C_{\text{min}}}{2} + \frac{C_{\text{max}} - C_{\text{min}}}{2} \cos \left( \frac{\pi}{w} z \right) \]

where \( w \) is the pitch width.
Capacitance Expressions

Example 2, overlapping area patterned electrodes:

\[
\frac{dC(z)}{dz} = -\frac{\pi}{2w} (C_{max} - C_{min}) \sin \left(\frac{\pi}{w} z\right)
\]

And:

\[
F_e = -\frac{\pi}{4w} (C_{max} - C_{min}) \sin \left(\frac{\pi x}{w}\right) (V - V_s)^2
\]
ELECTROSTATIC HARVESTER
DEVICES
Rotational Harvester - Boland - 2005

Another Rotational Harvester – Nakano and Suzuki - 2015

Nakano, Komori, Hattori, Suzuki, PowerMEMS 2015

Figure 1. Schematic of rotational electret energy harvester.

Figure 4. MEMS rotational electret energy harvester. A ball bearing is successfully installed its housing etched into the Si substrate. a) Rotor and stator substrate with a ball bearing, b) Rotational energy harvester stored in a plastic package.
Vibration Harvester - Y. Suzuki

**Figure 11.** In-plane electret generator with a high-aspect-ratio parylene spring.

Suzuki et. al. JM&M, 2010

**Figure 14.** Backside of the top Si structure. (a) Overview, (b) high-aspect-ratio parylene spring and (c) SEM image of the parylene spring.

**Figure 15.** Bottom substrate. (a) Overview, (b) magnified view of the electret and the electrodes in the dual-phase arrangement.
OMRON device has among the highest reported effectiveness \( E_H = \frac{P}{P_{VDRG}} \), of around 30-40%.

http://techon.nikkeibp.co.jp/english/NEWS_EN/20081117/161303/
Vibration Harvester - Boisseau

Ferrofluidic Electrostatic – Galchev

Galchev, Raz, and Paul, PowerMEMS 2012
Ferrofluidic Electrostatic – Kruupenkin

Krupenkin and Taylor, Nature Communications, 2012
Summary

• Electrostatic energy harvesters based on an air gap, with no electret have low energy density
• Electret based harvesters have energy densities on par with electromagnetic and piezoelectric
• Well suited to MEMS implementation
• Voltages high enough to be easily conditioned
• Fabrication and balancing of electrostatic forces can be a challenge