Introduction to Vibration Energy Harvesting

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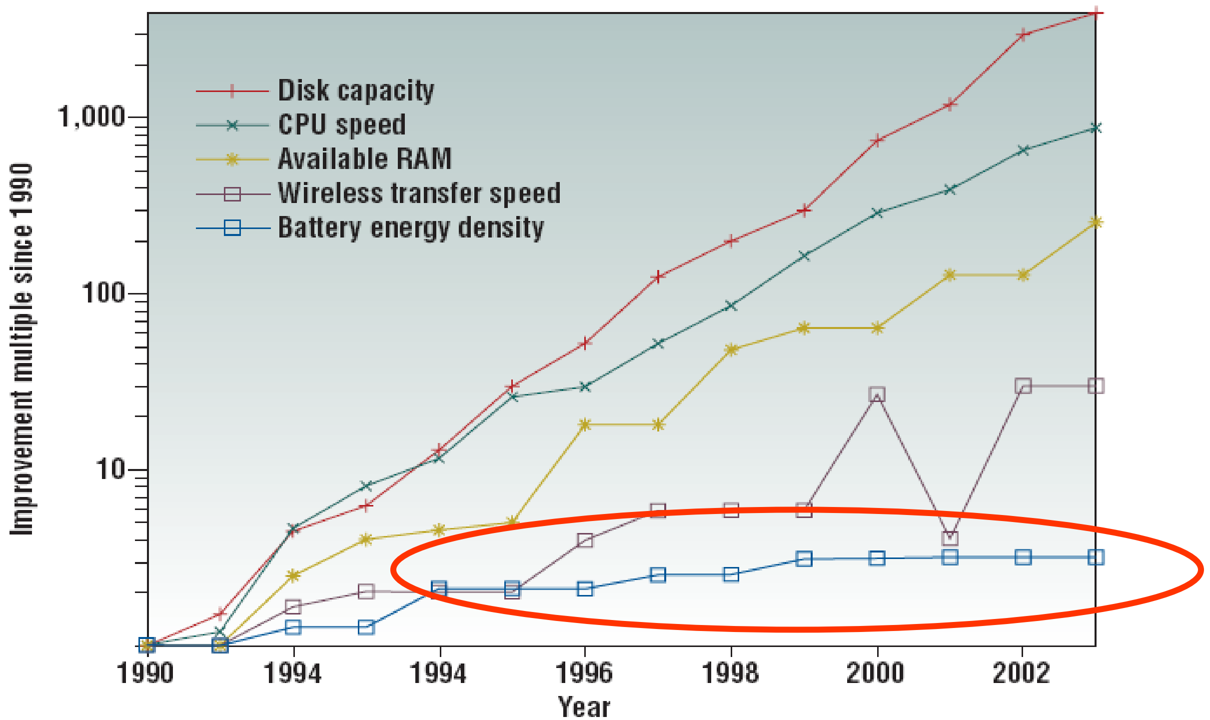
Summary

- Motivations of vibration energy harvesting
- State of the art and potential applications
- Vibration-to-electricity conversion methods
- Performance metrics
- Technical challenges and limits
- Conclusions

Beyond the batteries

- Moore's Law: transistors doubling every one or two years!
- Batteries power density and lifespan are limited

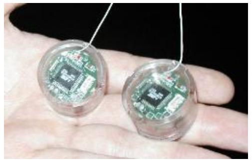


Mobile Computing Improvement – Paradiso, *et al.* Pervasive Computing, IEEE, 2005.

Beyond the batteries

Laptop

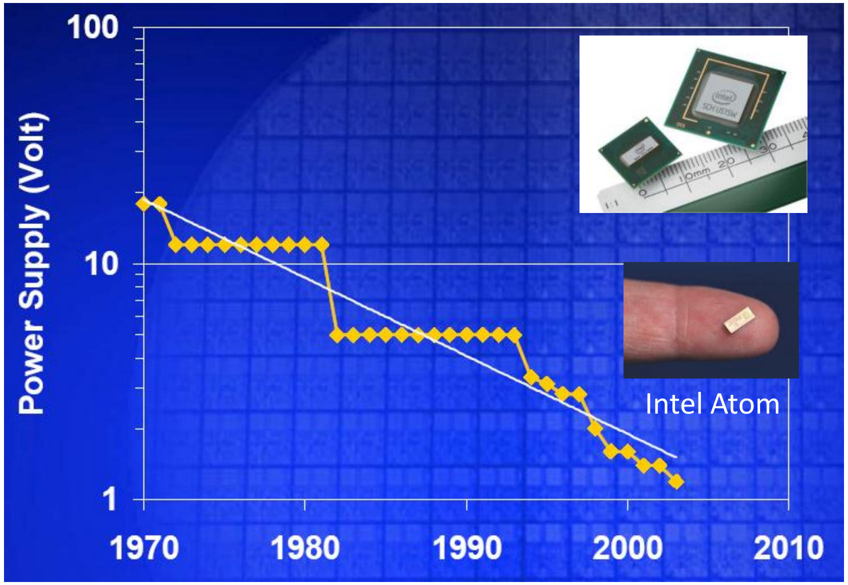
Smartphone



Wireless sensor nodes

Wearable sensors

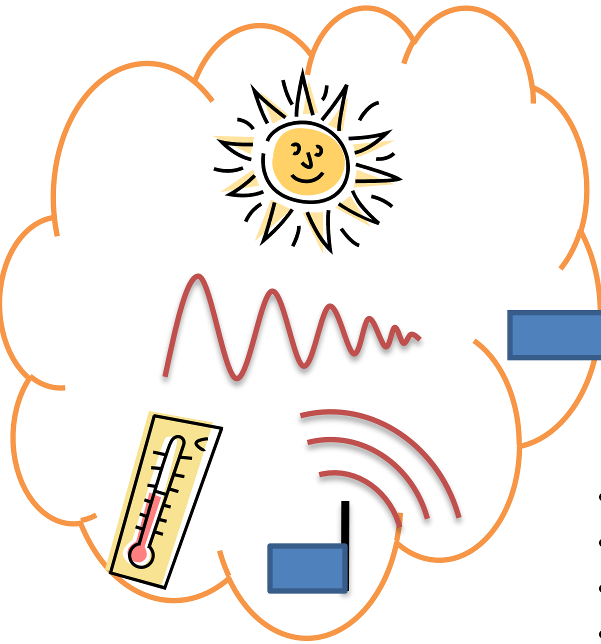
for size of **<1cm³** the power consumption goal is below **100 µW**



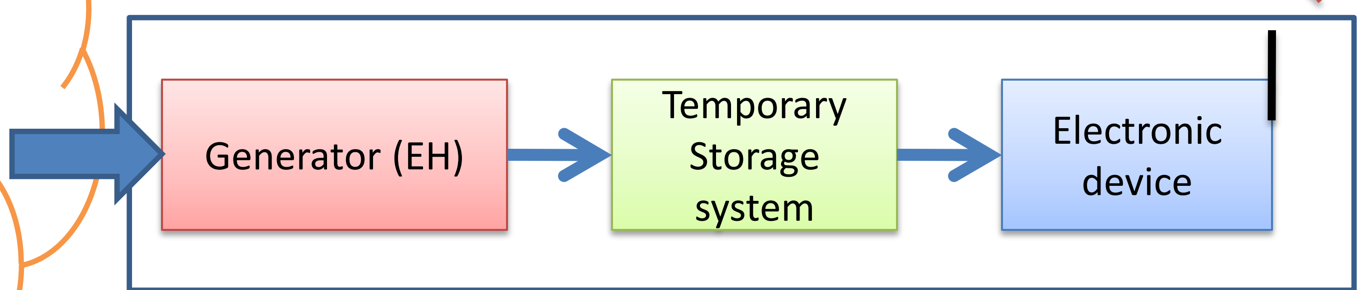
Intel - Moore, G.E., *“No exponential is forever: but "Forever" can be delayed!”*
Solid-State Circuits Conference, 2003

- Computing devices are becoming ubiquitous and pervasive!
- Power requirements must be scaled down, for size of <1cm^3 the power consumption goal is below 100 µW
- Problem: batteries must be recharged/replaced and eventually disposed

Energy harvesting as alternative for micropowering



NO NEED TO REPLACE BATTERIES!



- **Piezoelectric**
- **Electrodynamics**
- Photovoltaic
- Thermoelectric

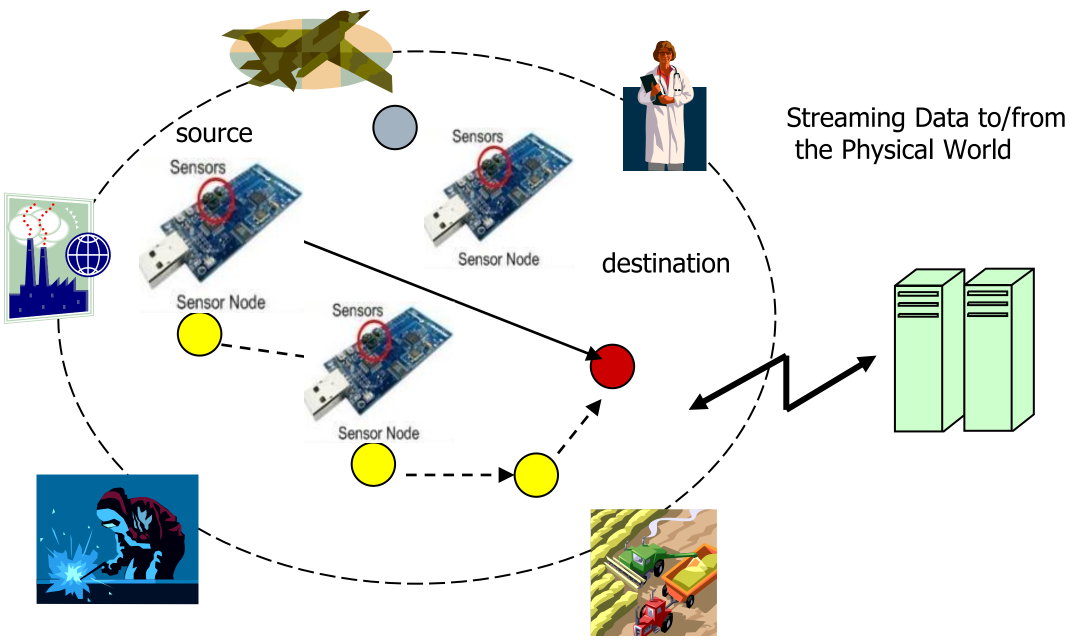
- Ultra capacitors
- Rechargeable Batteries

- Low power devices
- Wireless Sensors
- MEMS actuators
- Consumer electronics

Power sources

- **Photons**: Light , Infrared, Radio Frequencies
- **Kinetic**: vibrations, human motion, wind, hydro
- **Thermal**: temperature gradients
- **Biochemical**: glucose, metabolic reactions

Energy harvesting for Wireless Sensor Networks



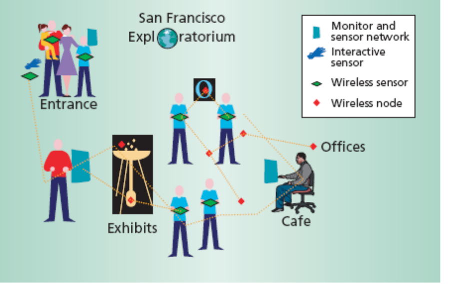
WSNs features

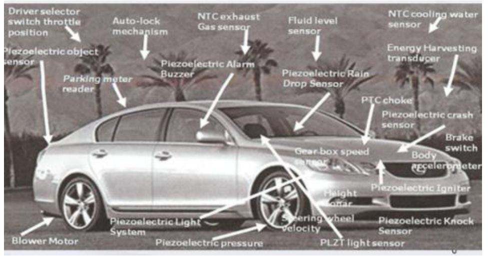
- Self-organizing
- Self-healing
- Pervasive
- Nearly invisible
- **Self-powering ?**

WSNs have vast applications

The main challenge for WSNs is to be
SELF-POWERING!!

- **Environmental Monitoring**
  - Habitat Monitoring (light, temperature, humidity)
  - Integrated Biology
- **Structural Monitoring**
- **Interactive and Control**
  - RFID, Real Time Locator, TAGS
  - Building, Automation
  - Transport Tracking, Cars sensors
- **Surveillance**
  - Pursuer-Evader
  - Intrusion Detection
  - Interactive museum exhibits
- **Medical remote sensing**
  - Emergency medical response
  - Monitoring, pacemaker, defibrillators
- **Military applications and Aerospace**





Benefits of Energy Harvesting

- Long lasting operability
- No chemical disposal
- Cost saving
- Safety
- Maintenance free
- No charging points
- Inaccessible sites operability
- Flexibility
- Applications otherwise impossible

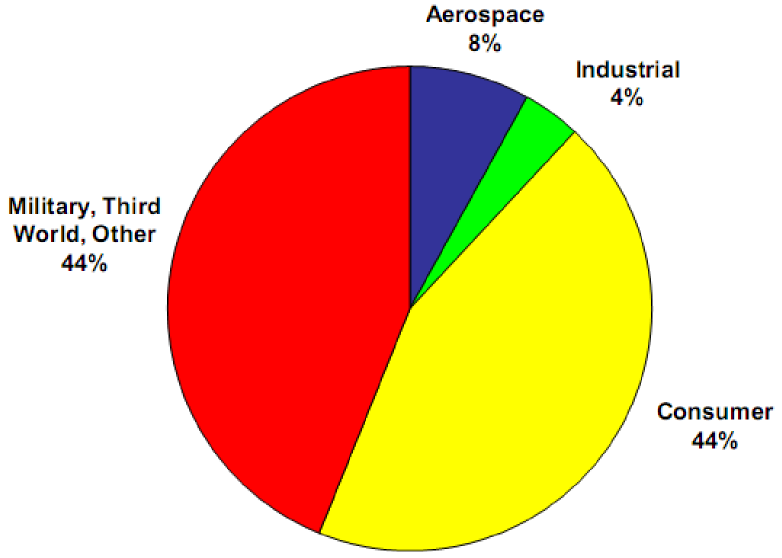
90% of WSNs cannot be enabled without Energy Harvesting technologies (solar, thermal, vibrations)

Market potential

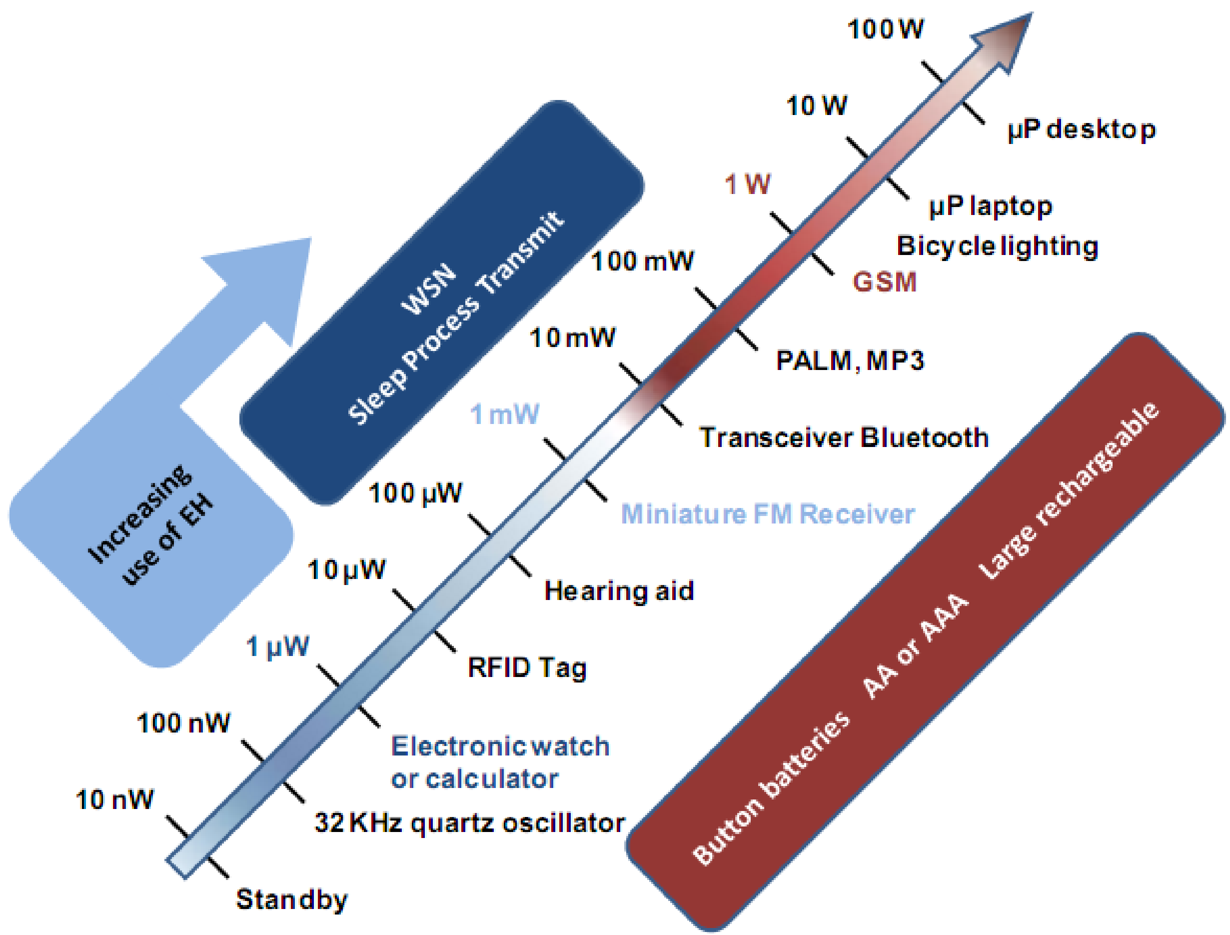
Market size of WSN's

- The overall wireless sensors market is estimate to grow to $4 billions by 2020 (Frost & Sullivan, 2006)
- The worldwide ULP market reached over 200 million units by 2010.
- Temperature monitoring and vibration spectra o sensitive plant equipment is growing recently around $22m (ARC Advisory Group, 2007).
- Vibration and velocity sensors and transmitters market is growing from $17.4m in 2006 to $112.5m in 2012 at a compound annual growth rate of 34.4%.

average annual growth rate of over 73%.



Power needs for small electronics



Source IDTechEx report "Energy Harvesting and Storage for Electronic Devices 2009-2019".

How much power is available from the ambient?

Texas Instruments, Energy Harvesting – White paper 2009

Energy Source	Harvested Power
Vibration/Motion	
Human	4 μW/cm²
Industry	100 μW/cm²
Temperature Difference	
Human	25 μW/cm²
Industry	1–10 mW/cm²
Light	
Indoor	10 μW/cm²
Outdoor	10 mW/cm²
RF	
GSM	0.1 μW/cm²
WiFi	0.001 mW/cm²

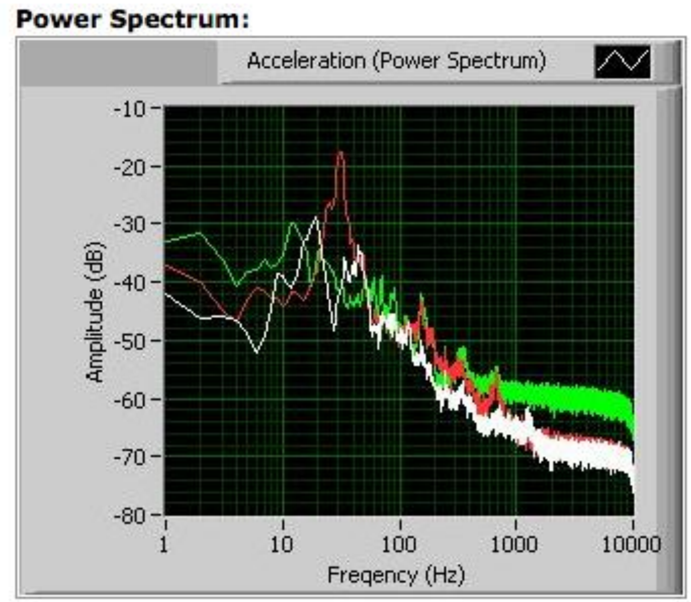
An average human walking up a mountain expends around 200 Watts of power.

The most amount of power your iPhone accepts when charging is 2.5 Watts.

How much power is available from the ambient?

Power source	Power (µW)/cm³	Energy (Joules)/cm³	Power (µW)/cm³/yr
Primary battery	N/A	2,880	90
Secondary battery	N/A	1,080	34
Micro fuel cell	N/A	3,500	110
Ultracapacitor	N/A	50–100	1.6–3.2
Heat engine	1×10^6	3,346	106
Radioactive (^{63}Ni)	0.52	1,640	0.52
Solar (outside)	15,000*	N/A	N/A
Solar (inside)	10*	N/A	N/A
Temperature	40*†	N/A	N/A
Human power	330	N/A	N/A
Air flow	380‡	N/A	N/A
Pressure variation	17§	N/A	N/A
Vibrations	375	N/A	N/A

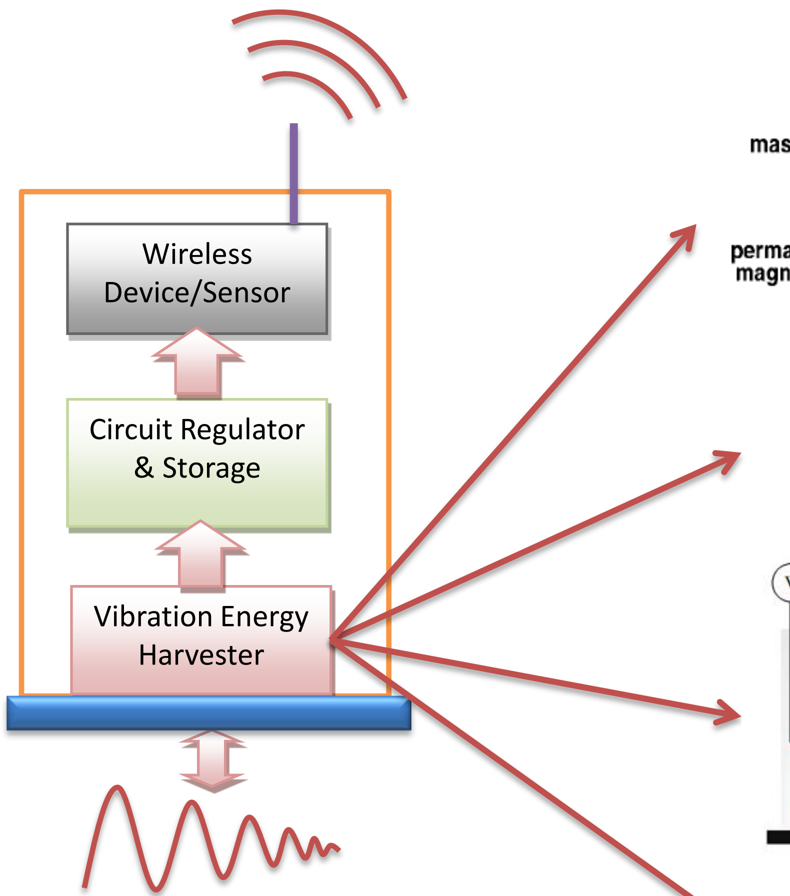
S. Roundy et al., Computing IEEE 2005.

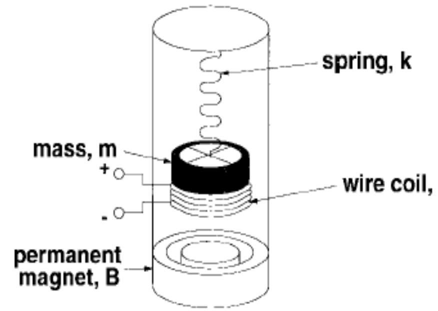


CAR Urban cycle. NIPS Lab Vibrations database

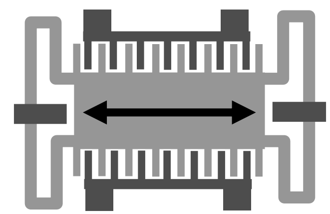
* Measured in power per square centimeter, rather than power per cubic centimeter.
† Demonstrated from a 5°C temperature differential.
‡ Assumes an air velocity of 5 m/s and 5 percent conversion efficiency.
§ Based on 1 cm³ closed volume of helium undergoing a 10°C change once a day.

Vibration energy harvesting

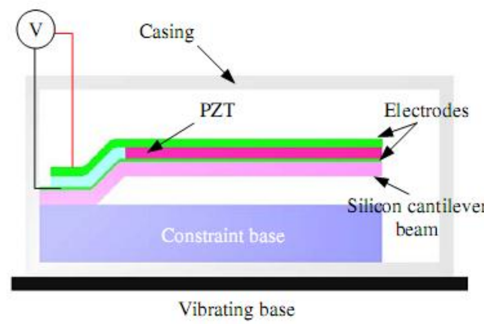




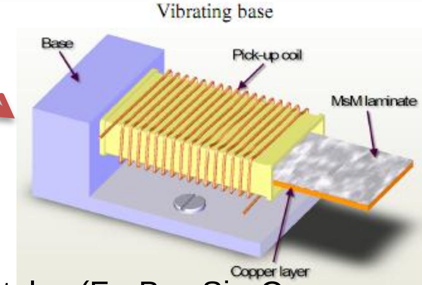
Electromagnetic



Electrostatic/Capacitive



Piezoelectric



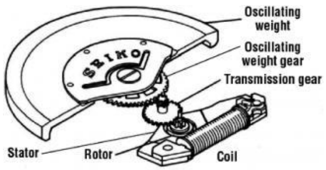
Magnetostrictive

- Ambient vibrations
- Human motion
- Wind, Hydro

crystalline alloy Terfenol-D
amorphous metallic glass Metglas ($Fe_8B_{13.5}Si_{3.5}C_2$).

Motion-driven powering applications

Well known → **Present** → **Forthcoming**

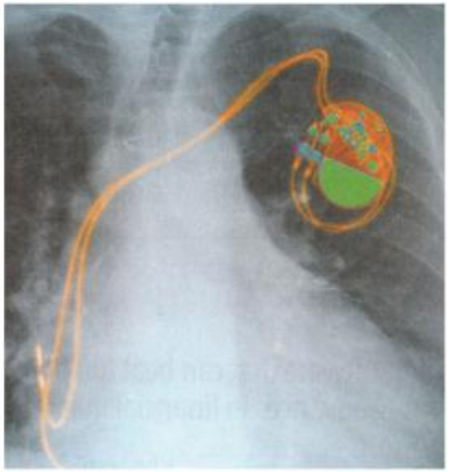


Self-charging Seiko wristwatch

Wind-up electrodynamic EH Torch, Dynamo

Perpetuum battery-less wireless sensing

- Enough energy for GPS, GSM transmissions from rotary pumps, trains, compressors
- Bearing, Vibration, Temperature, Air pollution monitoring
- Cargo monitoring and tracking
- Wireless bridge monitoring
- Active tags for Real Time Tracking and Logistics
- Oil refinery monitoring

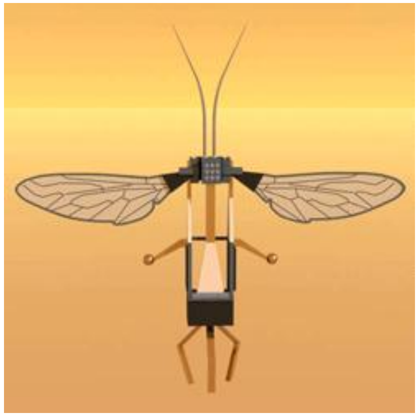


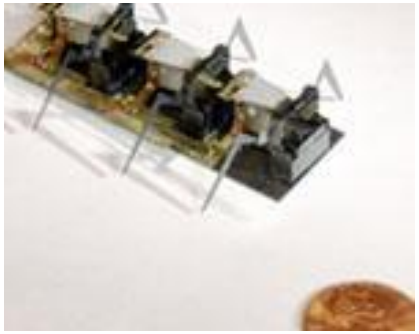
University of Southampton electrodynamic energy harvesting to run pacemaker and defibrillator

- **Medical implantations**
- **Medical remote sensing**
- **Body Area Network**

Motion-driven powering applications

Microrobotic Insects at Harvard Microrobotic Lab



A 1mm nanorobot flying at v=1 m/sec requires F ~ 4 microN and **P ~ 41 microW.**
[A. Freitas Jr., Nanomedicine, Volume, Landes Bioscience, 1999]

The input power for a 20mg robotic fly ranges between 10 – 100 microW depending on many factors: air friction, aerodynamic efficiency etc.

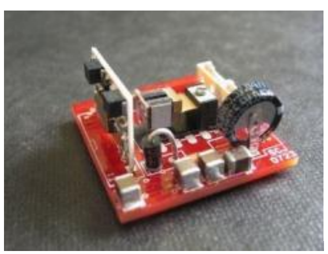
State of the art
macro to millimetric generators

Electrodynamic

Perpetuum PMG17 (England)

Up to 45mW @ 1g rms (15Hz)

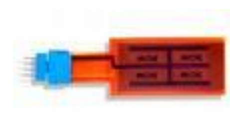
nPower® PEG



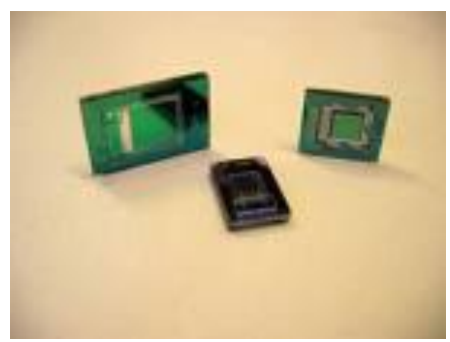
Micro-electromagnetic generator
S. Beeby 2007, (UK)

Piezoelectric

Mide' Volture (USA)
5mW @ 1grms (50Hz)

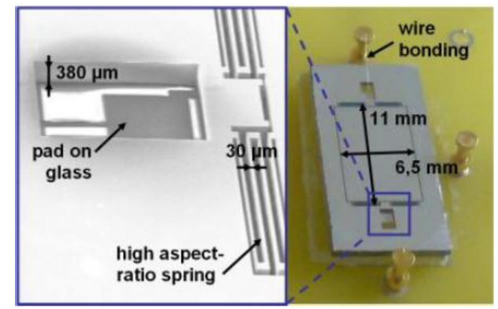


Holst-IMEC (Germany)
Micro PZ generator
500Hz 60uW @ 1g

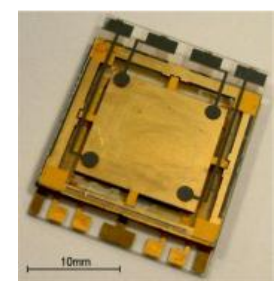


Electrostatic/Capacitive

ESIEE Paris – A. Mahmood Parracha



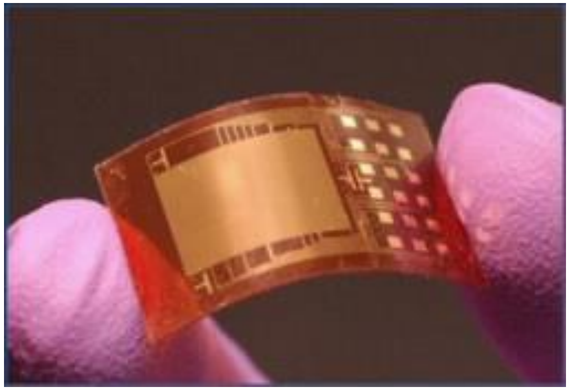
Imperial College, Mitcheson 2005 (UK)
Electrostatic generator 20Hz
2.5uW @ 1g



Microlab at UC Berkeley (Mitcheson)

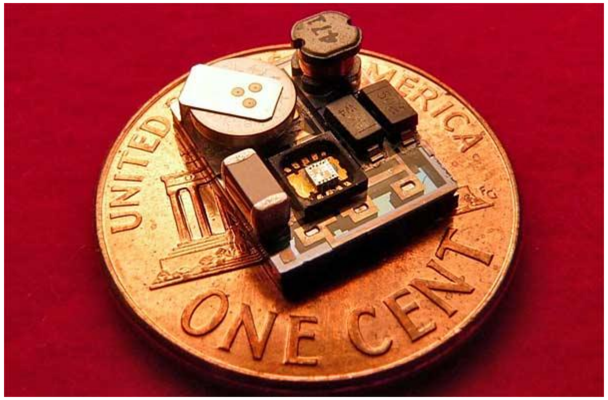
State of the art
micro- to nano- generators

zinc oxide (ZnO) nanowires

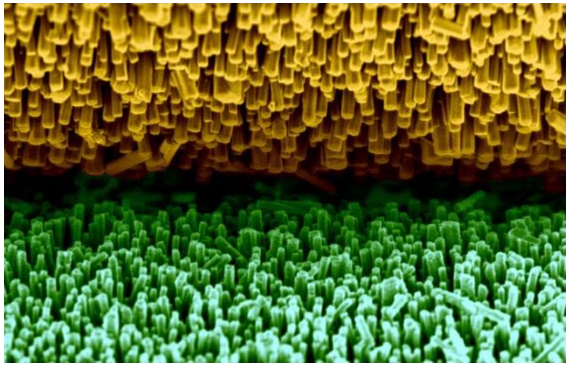


Zhong Lin Wang, Ph.D., Georgia Institute of Technology.

200 microwatts at 1.5g vibration @150Hz and charge an ultracapacitor to 1.85 volts.



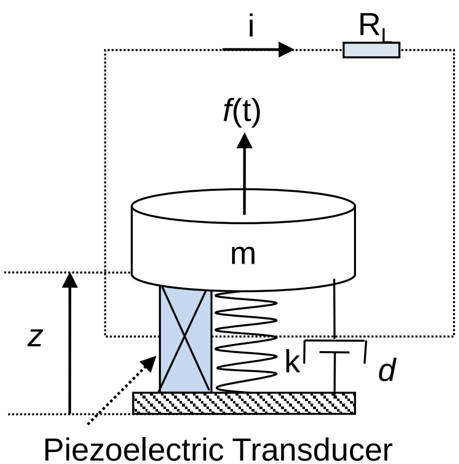
University of Michigan (USA)



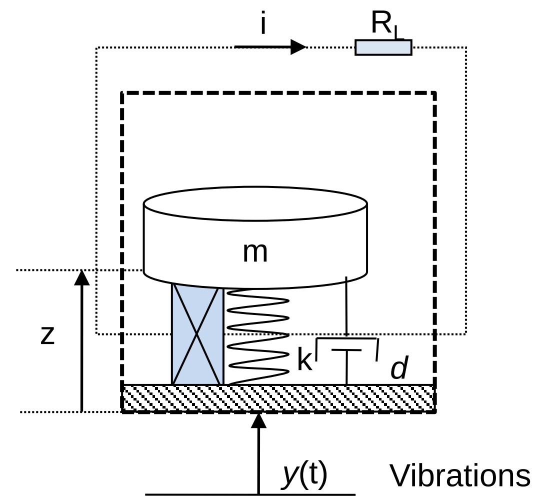
Nanogenerators produce electricity from running rodents

Vibration Energy Harvesters (VEHs): basic operating principles

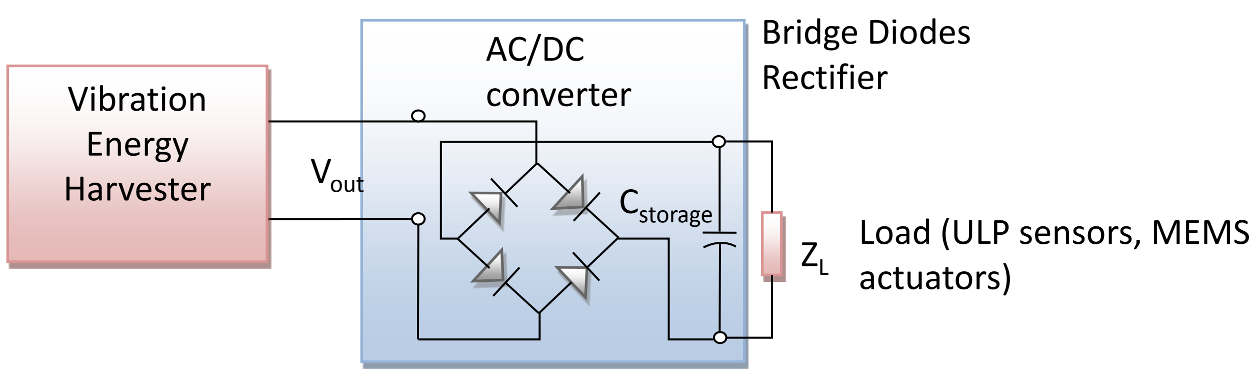
Direct force



Inertial force

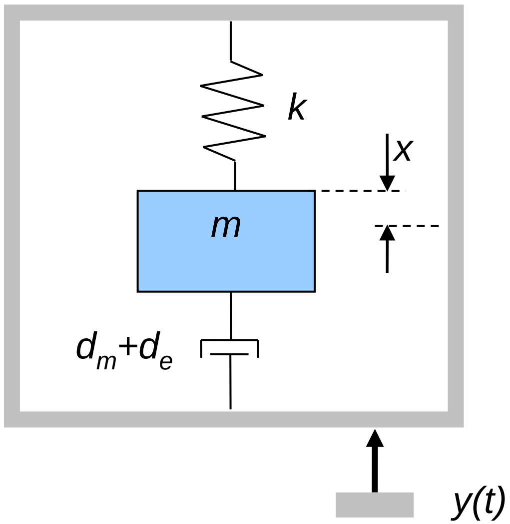


Inertial generators are more flexible than direct-force devices because they require only one point of attachment to a moving structure, allowing a greater degree of miniaturization.



Vibration Energy Harvesters (VEHs): basic operating principles

1-DOF generic mechanical-to-electrical conversion model [William & Yates]



Motion equation

$$m\ddot{x}(t)+(d_m+d_e)\dot{x}(t)+kx(t)=-m\ddot{y}(t)$$

Inertial force

$$f(t)=-m\ddot{y}=Y_0\sin(\omega t)$$

$$x(t)=\frac{\omega^2}{\sqrt{\left(\frac{k}{m}-\omega^2\right)^2+\left(\frac{(d_e+d_m)\omega}{m}\right)^2}}Y_0\sin(\omega t-\phi)$$

Steady state solution

setting $d_T=d_m+d_e$ the total damping coefficient, the phase angle ϕ is given by

$$\phi=\tan^{-1}\left(\frac{d_T\omega}{k-\omega^2 m}\right)$$

and the natural frequency $\omega_n=\sqrt{k/m}$

The instantaneous kinetic power $p(t)=-m\ddot{y}(t)[\dot{y}(t)+\dot{x}(t)]$ taking the Laplace transform of motion equation

$$H_{xf}(\omega)=\frac{X(\omega)}{Y(\omega)}=\frac{\omega^2}{-\omega^2+2i\omega(\zeta_e+\zeta_m)\omega_n+\omega_n^2}$$

Vibration Energy Harvesters (VEHs): basic operating principles

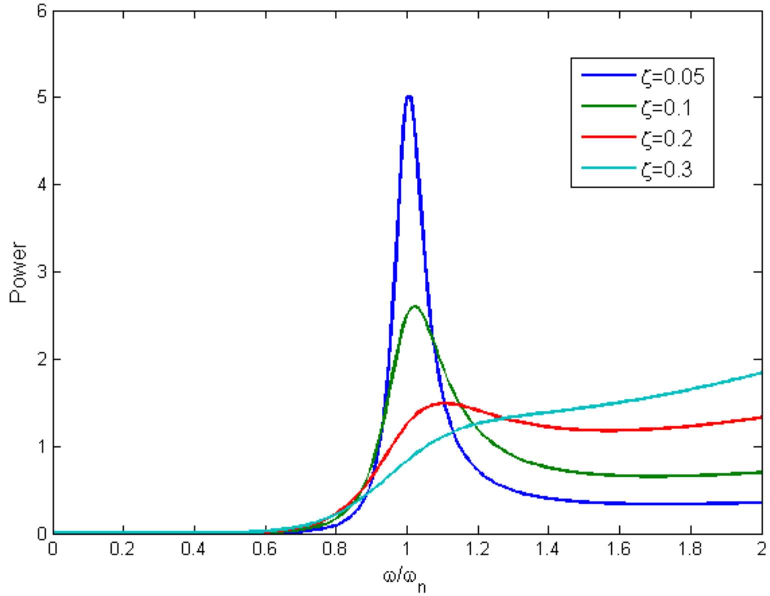
1-DOF generic mechanical-to-electrical conversion model [William & Yates]

the power dissipated by total electro-mechanical damping ratio, namely $\zeta_T=(\zeta_e+\zeta_m)=d_T/2m\omega_n$, is expressed by

$$P_{diss}(\omega)=m\zeta_T\omega_n\left|\dot{X}\right|^2=m\zeta_T\omega_n\omega^2\left|f\cdot H_{xf}\right|^2$$

that is

$$P_{diss}=\frac{m\zeta_T Y_0^{\,2}\left(\dfrac{\omega}{\omega_n}\right)^3\omega^3}{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta_T\left(\dfrac{\omega}{\omega_n}\right)\right]^2}$$



At natural resonance frequency, that is $\omega=\omega_n$, the maximum power is given by

$$P_{diss}=\frac{mY_0^{\,2}\omega_n^3}{4\zeta_T}$$ or with acceleration amplitude $A_0=\omega_n^2Y_0$. $\longrightarrow$ $$P_{diss}=\frac{mA_0^2}{4\omega_n\zeta_T}$$

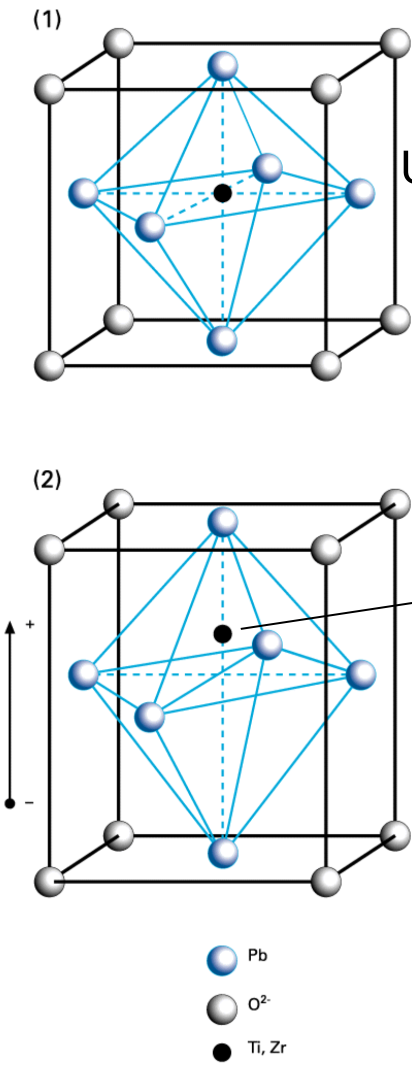
Separating parasitic damping ζ_m and transducer damping ζ_e for a particular transduction mechanism forced at natural frequency ω_n, the power can be maximized from the equation

$$P_{el}=\frac{m\zeta_e A^2}{4\omega_n(\zeta_m+\zeta_e)^2}$$

when the condition $\boldsymbol{\zeta_e=\zeta_m}$ is verified

Piezoelectric conversion

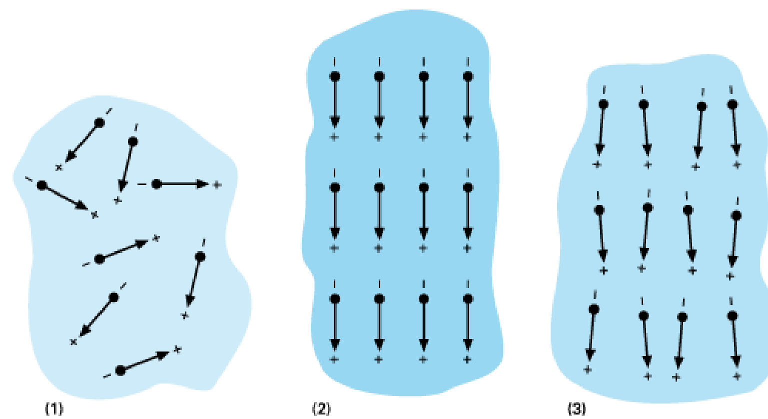
Piezoelectric materials



Unpolarized Crystal

Polarized Crystal

Pioneering work on the direct piezoelectric effect (stress-charge) in this material was presented by Jacques and Pierre Curie in 1880

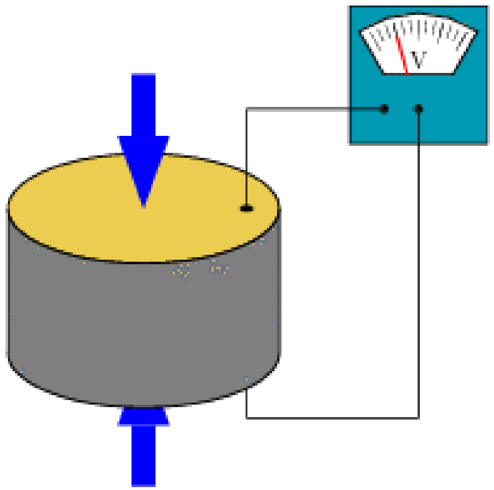


After poling the zirconate-titanate atoms are off center. The molecule becomes elongated and polarized

Piezoelectric conversion

Piezoelectric materials

Stress-to-charge conversion



direct piezoelectric effect

Naturally-occurring crystals

- Berlinite ($AlPO_4$), a rare phosphate mineral that is structurally identical to quartz
- Cane sugar
- Quartz
- Rochelle salt

Man-made ceramics

- Barium titanate ($BaTiO_3$)—Barium titanate was the first piezoelectric ceramic discovered.
- Lead titanate ($PbTiO_3$)
- Lead zirconate titanate ($Pb[Zr_xTi_{1-x}]O_3$ $0 \le x \le 1$)—more commonly known as ***PZT***, lead zirconate titanate is the most common piezoelectric ceramic in use today.
- Lithium niobate ($LiNbO_3$)

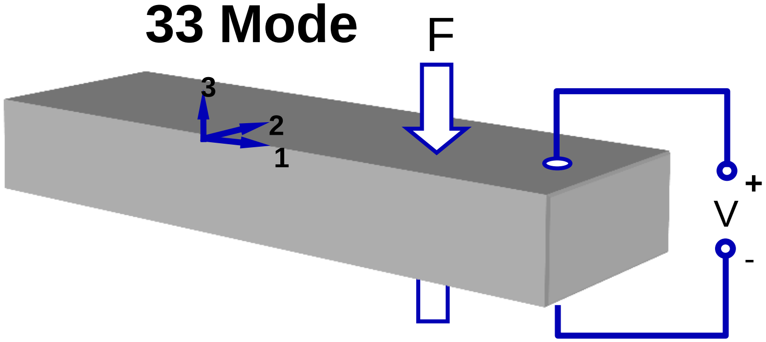
Polymers

- Polyvinylidene fluoride (PVDF): exhibits piezoelectricity several times greater than quartz. Unlike ceramics, long-chain molecules attract and repel each other when an electric field is applied.

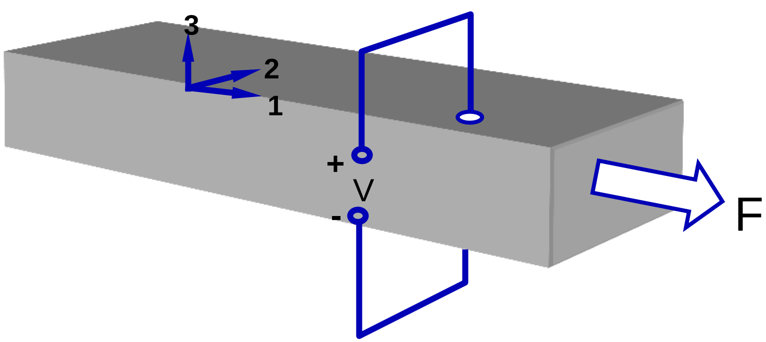
Piezoelectric conversion

Costitutive equations

33 Mode



31 Mode



$$S = [s_E]T + [d^t]E$$
$$D = [d]T + [\varepsilon_T]E$$

Strain-charge

$$T = [c^E]S - [e^t]E$$
$$D = [e]S + [\varepsilon^S]E$$

Stress-charge

- S = strain vector (6x1) in Voigt notation
- T = stress vector (6x1) [N/m^2]
- s_E = compliance matrix (6x6) [m^2/N]
- c^E = stifness matrix (6x6) [N/m^2]

- d = piezoelectric coupling matrix (3x6) in Strain-Charge [C/N]

- D = electrical displacement (3x1) [C/m^2]

- e = piezoelectric coupling matrix (3x6) in Stress-Charge [C/m^2]
- ε = electric permittivity (3x3) [F/m]

- E = electric field vector (3x1) [N/C] or [V/m]

Piezoelectric conversion

Costitutive equations

converse piezoelectric effect

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\ s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E = 2\left(s_{11}^E - s_{12}^E\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

direct piezoelectric effect

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Depending on the independent variable choice 4 piezoelectric coefficients are defined:

$$d_{ij} = \left(\frac{\partial D_i}{\partial T_j}\right)^E = \left(\frac{\partial S_j}{\partial E_i}\right)^T$$

$$e_{ij} = \left(\frac{\partial D_i}{\partial S_j}\right)^E = -\left(\frac{\partial T_j}{\partial E_i}\right)^S$$

$$g_{ij} = -\left(\frac{\partial E_i}{\partial T_j}\right)^D = \left(\frac{\partial S_j}{\partial D_i}\right)^T$$

$$h_{ij} = -\left(\frac{\partial E_i}{\partial S_j}\right)^D = -\left(\frac{\partial T_j}{\partial D_i}\right)^S$$

Voigt notation is used to represent a symmetric tensor by reducing its order.
Due to the symmetry of the stress tensor, strain tensor, and stiffness tensor, only 21 elastic coefficients are independent. *S* and *T* appear to have the "vector form" of 6 components. Consequently, *s* appears to be a 6 by 6 matrix instead of rank-4 tensor.

Piezoelectric conversion

Material properties example

Property	PZT-5H	PZT-5A	$BaTiO_3$	PVDF
d_{33} (10^{-12} C N^{-1})	593	374	149	−33
d_{31} (10^{-12} C N^{-1})	−274	−171	78	23
g_{33} (10^{-3} V m N^{-1})	19.7	24.8	14.1	330
g_{31} (10^{-3} V m N^{-1})	−9.1	−11.4	5	216
k_{33}	0.75	0.71	0.48	0.15
k_{31}	0.39	0.31	0.21	0.12
Relative permittivity ($\varepsilon/\varepsilon_o$)	3400	1700	1700	12

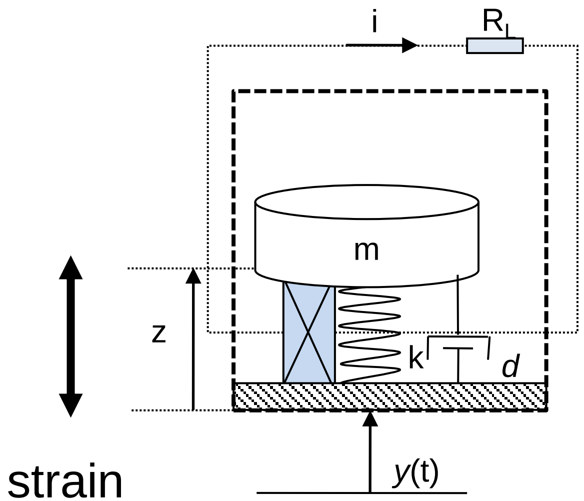
Electromechanical Coupling is an adimensional factor defined as the ratio between the **mechanical energy converted** and the **electric energy input** or the **electric energy converted** per **mechanical energy input**

$$k_{31}^2 = \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T}$$

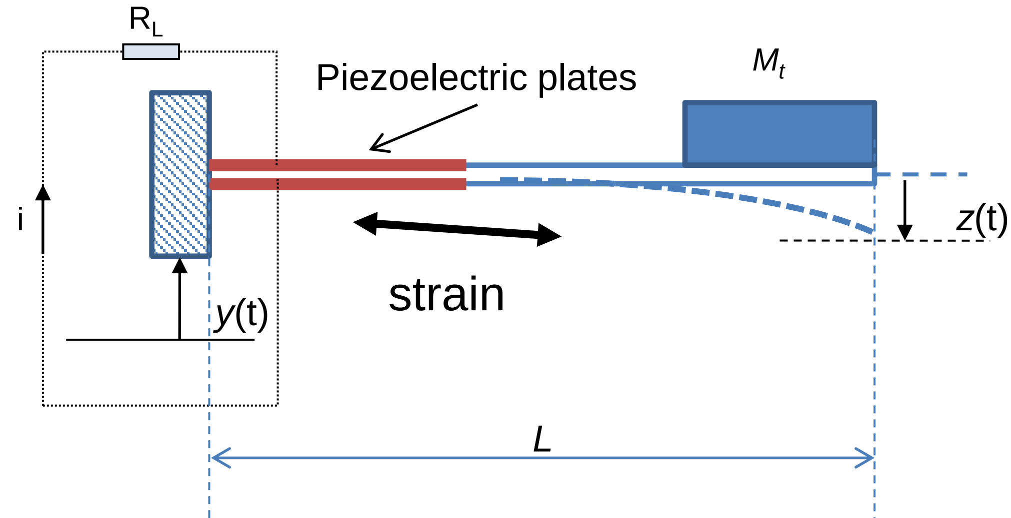
Piezoelectric conversion

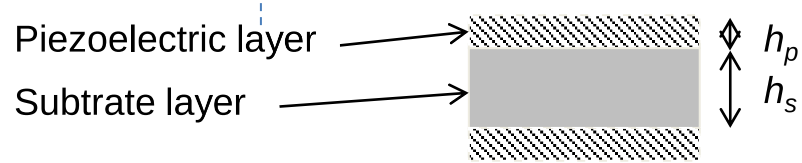
Mechanical-to-electrical conversion models

Piezoelctric bulk (33 mode)

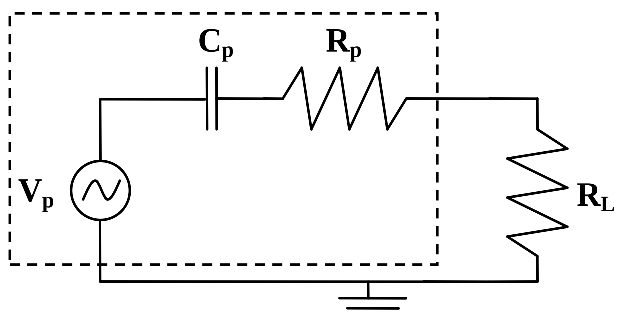


Cantilever beam (31 mode)





Piezoelectric generator



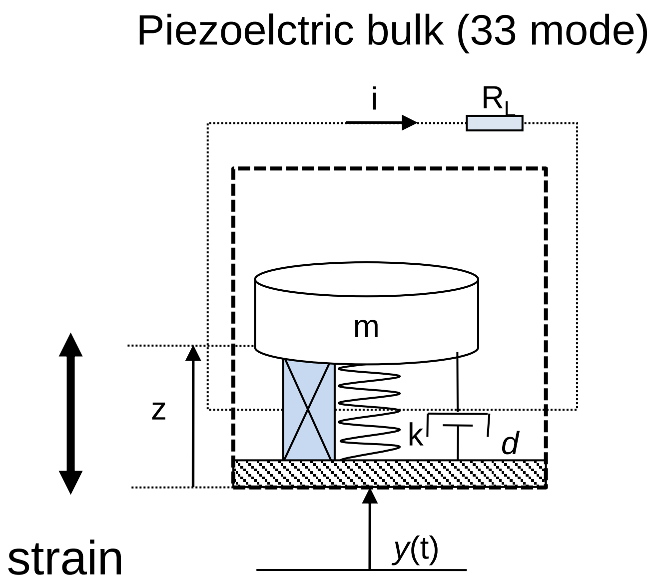
At open circuit $$V_{oc} = -\frac{d_{31} \cdot h}{\varepsilon^S} T_1$$

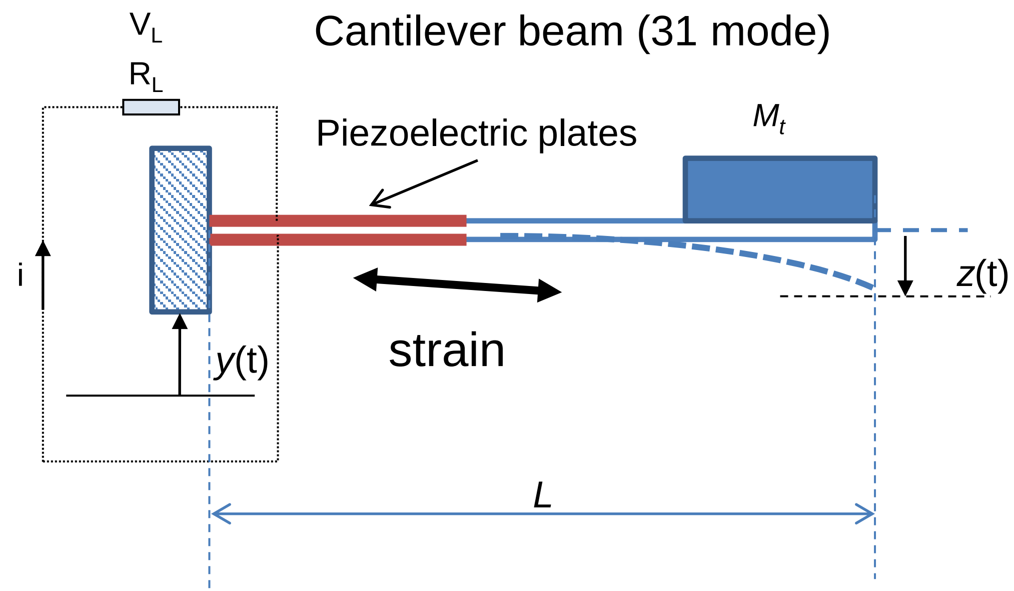
The power delivered to the load is simply $$P = \frac{V_L^2}{R_L}$$

S. Roundy, Energy scavenging for wireless sensor networks, Kluwer

Piezoelectric conversion

Mechanical-to-electrical conversion models





$$f_0 = \frac{1}{4L_3\sqrt{\rho s_{33}^D}}$$

Natural undamped frequency

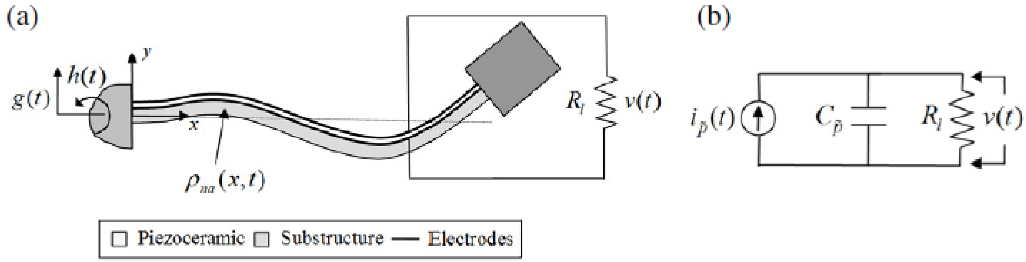
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \qquad k = \frac{Eh^3b}{4L^3}$$

$$I = \frac{bh^3}{12}$$

Inertia moment

Piezoelectric conversion

Coupled distribuited parameter model for cantilever beam VEH [Erturk and Inmann (2008)]



$$T_1 = c_{11}^E S_1 - e_{31} E_3,$$

$$D_3 = e_{31} S_1 + \varepsilon_{33}^S E_3,$$

Free vibration of the cantilever beam is governed by

$$\frac{\partial^2 M(x,t)}{\partial x^2} + c_s I \frac{\partial^5 w_{\mathrm{rel}}(x,t)}{\partial x^4\,\partial t} + c_a \frac{\partial w_{\mathrm{rel}}(x,t)}{\partial t} + m \frac{\partial^2 w_{\mathrm{rel}}(x,t)}{\partial t^2} = -[m + M_t \delta(x-L)] \frac{\partial^2 w_b(x,t)}{\partial t^2}$$

Internal strain rate (Kelvin Voigt) damping term

Air viscous damping

The absolute transverse motion of the beam at any point *x* and time *t* can be written as

$$w(x,t) = w_b(x,t) + w_{\mathrm{rel}}(x,t)$$

$$w_b(x,t) = g(t) + x h(t)$$

The internal bending is

$$M(x,t) = -b\left(\int_{-h_{\tilde{p}}-h_{\tilde{s}}/2}^{-h_{\tilde{s}}/2} T_1^{\tilde{p}}\, y\, dy + \int_{-h_{\tilde{s}}/2}^{h_{\tilde{s}}/2} T_1^{\tilde{s}}\, y\, dy + \int_{h_{\tilde{s}}/2}^{h_{\tilde{p}}+h_{\tilde{s}}/2} T_1^{\tilde{p}}\, y\, dy\right)$$

The axial strain at a certain is simply proportional to the curvature of the beam at that position (x):

$$S_1(x,y,t) = -y \frac{\partial^2 w_{\mathrm{rel}}(x,t)}{\partial x^2}.$$

Erturk, A. and D. J. Inman (2008). "A distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters." Journal of vibration and acoustics **130**

Piezoelectric conversion

Coupled distribuited parameter model for cantilever beam VEH

[Erturk and Inmann (2008)]

the coupled beam equation can be obtained for the series connection

$$YI\frac{\partial^4 w_{\mathrm{rel}}^{\mathrm{s}}(x,t)}{\partial x^4} + c_{\mathrm{s}} I \frac{\partial^5 w_{\mathrm{rel}}^{\mathrm{s}}(x,t)}{\partial x^4 \partial t} + c_{\mathrm{a}} \frac{\partial w_{\mathrm{rel}}^{\mathrm{s}}(x,t)}{\partial t} + m\frac{\partial^2 w_{\mathrm{rel}}^{\mathrm{s}}(x,t)}{\partial t^2} + \vartheta_{\mathrm{s}} v_{\mathrm{s}}(t)\left[\frac{\mathrm{d}\delta(x)}{\mathrm{d}x} - \frac{\mathrm{d}\delta(x-L)}{\mathrm{d}x}\right] = -[m + M_{\mathrm{t}}\delta(x-L)]\frac{\partial^2 w_{\mathrm{b}}(x,t)}{\partial t^2}$$

vibration response relative to the base can be expressed by using the modal expansion theorem

$$w_{\mathrm{rel}}^{\mathrm{s}}(x,t) = \sum_{r=1}^{\infty} \phi_r(x)\eta_r^{\mathrm{s}}(t),$$

With mass normalized eigenfunctions $\phi_r(x) = C_r\left[\cos\frac{\lambda_r}{L}x - \cosh\frac{\lambda_r}{L}x + \varsigma_r\left(\sin\frac{\lambda_r}{L}x - \sinh\frac{\lambda_r}{L}x\right)\right]$ with eigenvalues λ_r

modal amplitude Cr and eigenvalues λ_r
are evaluated by normalizing the eigenfunctions according to the orthogonality and bonduary conditions

the undamped natural frequency of the *r* th vibration mode in short circuit

$$\omega_r = \lambda_r^2\sqrt{\frac{YI}{mL^4}}$$

the electric current output is then obtained from the Gauss law

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_A \mathbf{D}\cdot\mathbf{n}\,\mathrm{d}A\right) = \frac{v(t)}{R_{\mathrm{l}}} \Rightarrow \frac{\bar{\varepsilon}_{33}^S bL}{h_{\bar{\mathrm{p}}}}\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{v(t)}{R_{\mathrm{l}}} = -\bar{e}_{31}h_{\bar{\mathrm{p}}\mathrm{c}}b\int_0^L \frac{\partial^3 w_{\mathrm{rel}}(x,t)}{\partial x^2\partial t}\mathrm{d}x$$

One can then substitute the modal expansion

$$\Rightarrow \frac{\bar{\varepsilon}_{33}^S bL}{h_{\bar{\mathrm{p}}}}\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{v(t)}{R_{\mathrm{l}}} = \sum_{r=1}^{\infty}\kappa_r\frac{\mathrm{d}\eta_r(t)}{\mathrm{d}t}$$

Where the modal coupling term in the electrical circuit is

$$\boxed{\kappa_r = -\bar{e}_{31}h_{\bar{\mathrm{p}}\mathrm{c}}b\int_0^L \frac{\mathrm{d}^2\phi_r(x)}{\mathrm{d}x^2}\mathrm{d}x = -\bar{e}_{31}h_{\bar{\mathrm{p}}\mathrm{c}}b\left.\frac{\mathrm{d}\phi_r(x)}{\mathrm{d}x}\right|_{x=L}}$$

segmented electrodes can be used in harvesting energy from the modes higher than the fundamental mode in order to avoid cancellations effects

Piezoelectric conversion

Coupled distribuited parameter model for cantilever beam VEH
[Erturk and Inmann (2008)]

Coupled electro-mechanical beam equations in modal coordinates

$$\boxed{\frac{\mathrm{d}^2\eta_r^{\mathrm{s}}(t)}{\mathrm{d}t^2} + 2\zeta_r\omega_r\frac{\mathrm{d}\eta_r^{\mathrm{s}}(t)}{\mathrm{d}t} + \omega_r^2\eta_r^{\mathrm{s}}(t) + \chi_r^s v_{\mathrm{s}}(t) = f_r(t)}$$

With the modal electromechanical coupling term $\chi_r^s = \vartheta_{\mathrm{s}}\left.\frac{\mathrm{d}\phi_r(x)}{\mathrm{d}x}\right|_{x=L}$

the modal mechanical forcing

$$f_r(t) = -m\left(\frac{\mathrm{d}^2 g(t)}{\mathrm{d}t^2}\int_0^L \phi_r(x)\,\mathrm{d}x + \frac{\mathrm{d}^2 h(t)}{\mathrm{d}t^2}\int_0^L x\phi_r(x)\,\mathrm{d}x\right) - M_{\mathrm{t}}\phi_r(L)\left(\frac{\mathrm{d}^2 g(t)}{\mathrm{d}t^2} + L\frac{\mathrm{d}^2 h(t)}{\mathrm{d}t^2}\right).$$

the Kirchhoff laws applied to the equivalent electrical circuit gives

$$\boxed{C_{\tilde{\mathrm{p}}}\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{v(t)}{R_{\mathrm{l}}} = i_{\tilde{\mathrm{p}}}(t)}$$

with

$$C_{\tilde{\mathrm{p}}} = \frac{\bar{\varepsilon}_{33}^S bL}{h_{\tilde{\mathrm{p}}}}, \qquad i_{\tilde{\mathrm{p}}}(t) = \sum_{r=1}^{\infty}\kappa_r\frac{\mathrm{d}\eta_r(t)}{\mathrm{d}t}$$

Piezoelectric conversion

Coupled distribuited parameter model for cantilever beam VEH
[Erturk and Inmann (2008)]

If the translational and rotational components of the base displacement are harmonic

$$g(t) = Y_0 e^{j\omega t},$$
$$h(t) = \vartheta_0 e^{j\omega t}$$

The modal forcing can be expressed

$$f_r(t) = F_r e^{j\omega t} \qquad F_r = \omega^2\left[m\left(Y_0\int_0^L \phi_r(x)\mathrm{d}x + \theta_0\int_0^L x\phi_r(x)\mathrm{d}x\right) + M_t\phi_r(L)(Y_0 + L\theta_0)\right]$$

The resulting voltage amplitude

$$v_s(t) = \frac{\sum_{r=1}^{\infty} \frac{\mathrm{j}\omega\kappa_r F_r}{\omega_r^2 - \omega^2 + \mathrm{j}2\zeta_r\omega_r\omega}}{\frac{1}{R_l} + \mathrm{j}\omega\frac{C_{\tilde{p}}}{2} + \sum_{r=1}^{\infty}\frac{\mathrm{j}\omega\kappa_r\chi_r^s}{\omega_r^2 - \omega^2 + \mathrm{j}2\zeta_r\omega_r\omega}} \mathrm{e}^{\mathrm{j}\omega t}$$

the steady state modal mechanical response

$$\eta_r^s(t) = \left(F_r - \chi_r^s \frac{\sum_{r=1}^{\infty} \frac{\mathrm{j}\omega\kappa_r F_r}{\omega_r^2 - \omega^2 + \mathrm{j}2\zeta_r\omega_r\omega}}{\frac{1}{R_l} + \mathrm{j}\omega\frac{C_{\tilde{p}}}{2} + \sum_{r=1}^{\infty}\frac{\mathrm{j}\omega\kappa_r\chi_r^s}{\omega_r^2 - \omega^2 + \mathrm{j}2\zeta_r\omega_r\omega}}\right) \times \frac{\mathrm{e}^{\mathrm{j}\omega t}}{\omega_r^2 - \omega^2 + \mathrm{j}2\zeta_r\omega_r\omega}.$$

Piezoelectric conversion

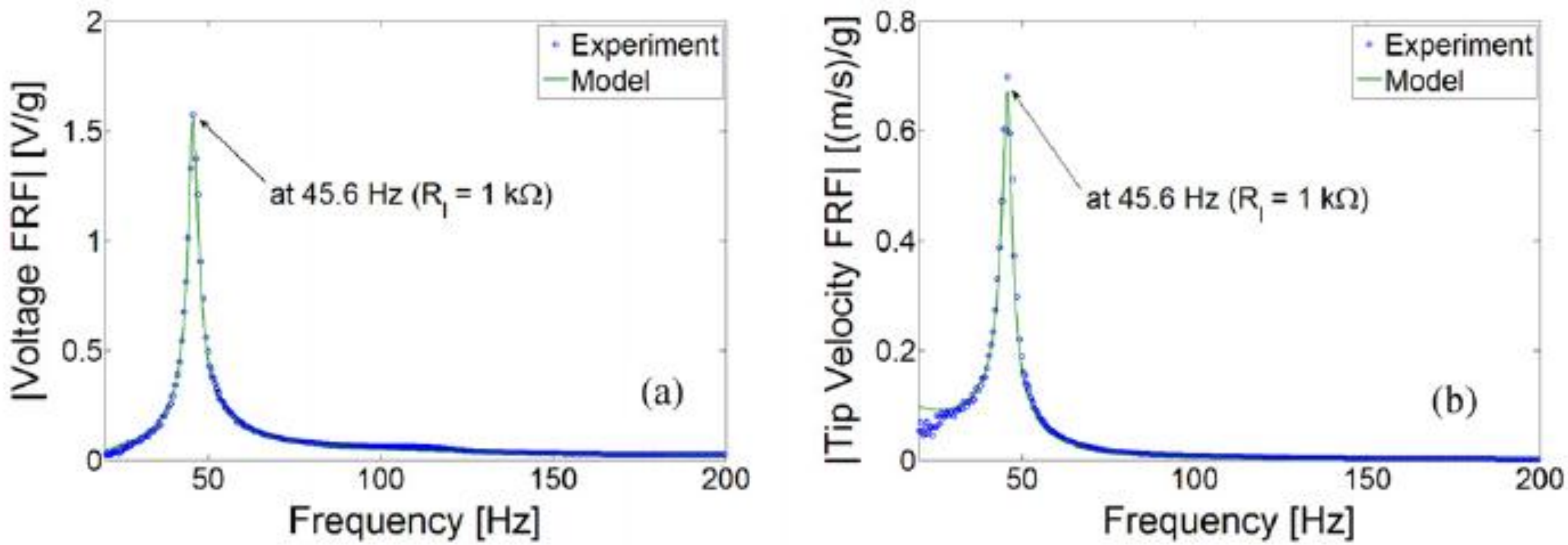
Coupled distribuited parameter model for cantilever beam VEH
[Erturk and Inmann (2008)]

The transverse displacement response relative to the base at point *x* on the beam is obtained

$$w_{\mathrm{rel}}^{s}(x,t)=\sum_{r=1}^{\infty}\left[\left(F_r-\chi_r^s\,\frac{\sum_{r=1}^{\infty}\frac{\mathrm{j}\omega\kappa_r F_r}{\omega_r^2-\omega^2+\mathrm{j}2\zeta_r\omega_r\omega}}{\frac{1}{R_{\mathrm{l}}}+\mathrm{j}\omega\frac{C_{\tilde{\mathrm{p}}}}{2}+\sum_{r=1}^{\infty}\frac{\mathrm{j}\omega\kappa_r\chi_r^s}{\omega_r^2-\omega^2+\mathrm{j}2\zeta_r\omega_r\omega}}\right)\times\frac{\phi_r(x)\mathrm{e}^{\mathrm{j}\omega t}}{\omega_r^2-\omega^2+\mathrm{j}2\zeta_r\omega_r\omega}\right]$$

electrical peak power for $\omega \simeq \omega_r$

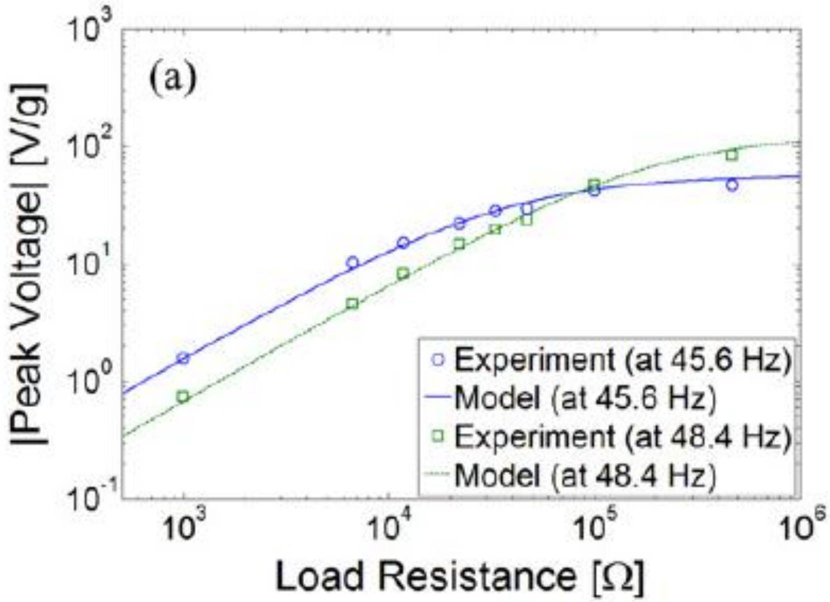
$$\boxed{\left|\hat{P}(t)\right|=\frac{R_l(\omega\varphi_r F_r)^2}{\left[\omega_r^2-\omega^2(1+2\zeta_r\omega_r R_l C_p)\right]^2+\left[2\zeta_r\omega_r\omega+\omega R_l\left[C_p\left(\omega_r^2-\omega^2\right)+\varphi_r\chi_r\right]\right]^2}}$$

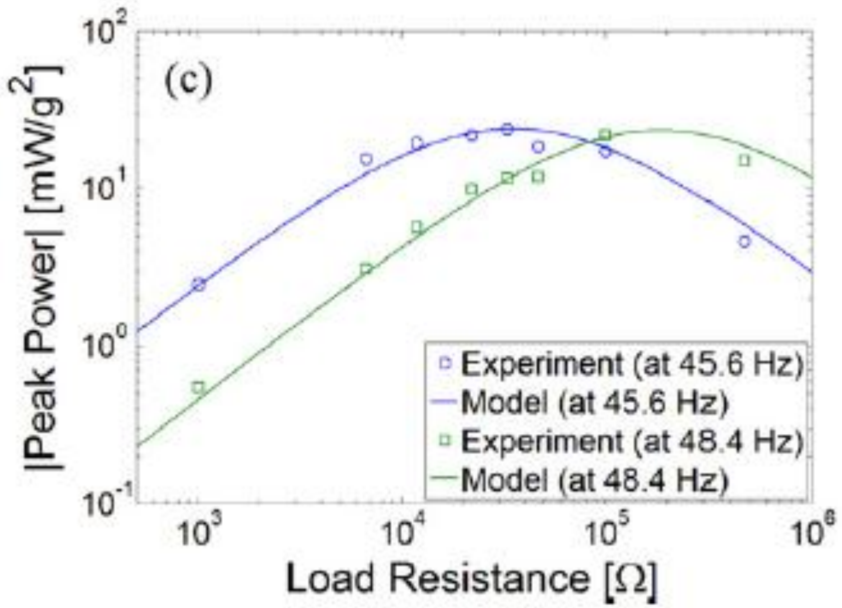


Piezoelectric conversion

Coupled distribuited parameter model for cantilever beam VEH

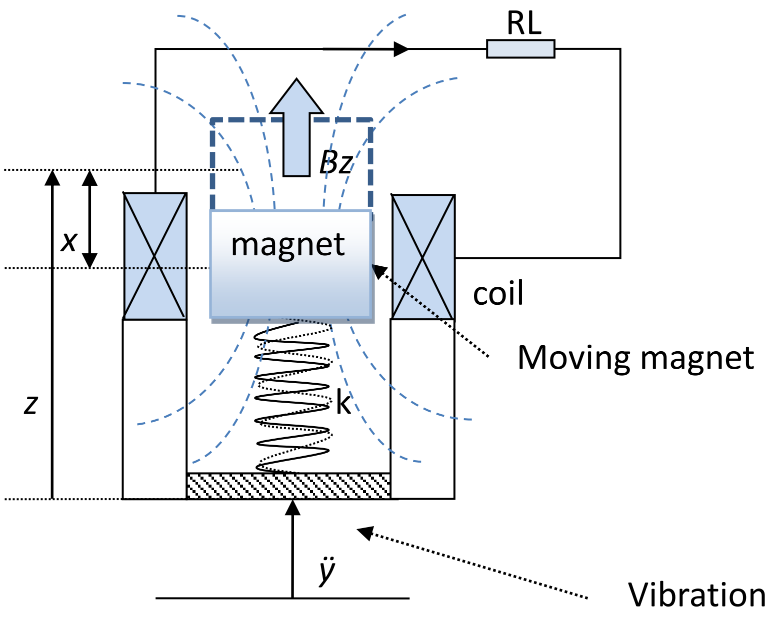
[Erturk and Inmann (2008)]





Erturk, A. and D. J. Inman (2009). "An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations." Smart materials and structures **18**: 025009.

Electromagnetic generators



The Faraday's law states that $\varepsilon = -\frac{d\Phi_B}{dt}$

for a coil moving through a perpendicular constant magnetic field, the maximum open circuit voltage across the coil is

$$V_{oc} = NBl\frac{dx}{dt}$$

N is the number of turns in the coil, *B* is the strength of the magnetic field, *l* is length of a winding and *x* is the relative displacement distance between the coil and magnet

The governing equations for only one-DOF model of a EM VEH can be written in a more general form *

$$\begin{cases} m\ddot{z} + d\dot{z} + kz = -\alpha V_L - m\ddot{y} \\ \dot{V}_L + \omega_c V_L = \delta_c \omega_c \dot{z} \end{cases}$$

Where

$\alpha = B_z l / R_L$ Electrical coupling force factor

$\delta_c = B_z l$ Conversion factor

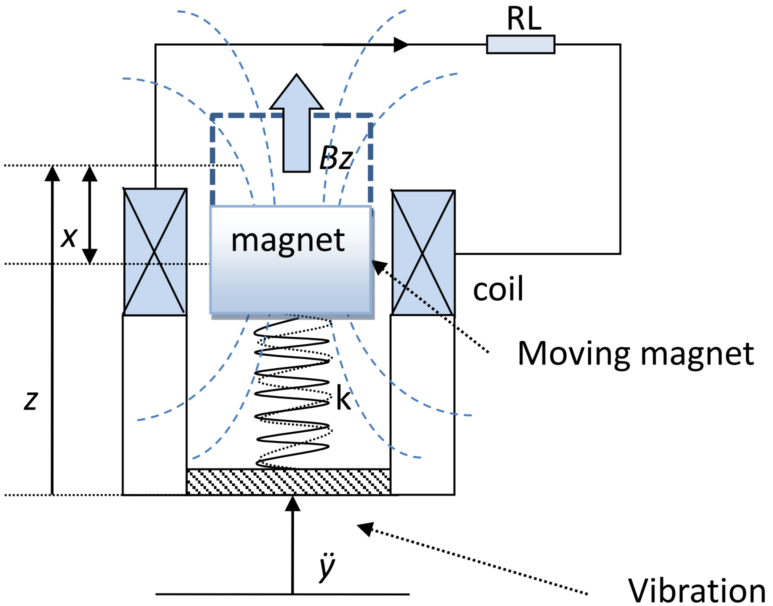
$\omega_c = R_L / L_e$ Characteristic cut-off frequency

$L_e = \mu_0 N^2 \pi R^2 / h_b$ Coil self-inductance

Joon Kim, K., F. Cottone, et al. (2010). "Energy scavenging for energy efficiency in networks and applications." Bell Labs Technical Journal **15**(2): 7-29.

Electromagnetic generators

Transfer functions



By transforming the motion equations and into Laplace domain with ***s*** as Laplace variable, considering only the forced solution, the acceleration of the base being *Y(s)*

$$\begin{pmatrix} ms^2+ds+k & \alpha \\ -\delta_c\omega_c s & s+\omega_c \end{pmatrix}\begin{pmatrix} Z \\ V \end{pmatrix}=\begin{pmatrix} -mY \\ 0 \end{pmatrix}$$

The left-side matrix A represents the generalized impedance of the oscillating system. So the solution is given by

$$Z=\frac{-mY}{\det A}(s+\omega_c)=\frac{-mY(s+\omega_c)}{ms^3+(m\omega_c+d)s^2+(k+\alpha\delta_c\omega_c+d\omega_c)s+k\omega_c}$$

$$V=\frac{-mY}{\det A}\delta_c\omega_c s=\frac{-mY\delta_c\omega_c s}{ms^3+(m\omega_c+d)s^2+(k+\alpha\delta_c\omega_c+d\omega_c)s+k\omega_c}$$

the transfer functions between displacement *Z*, voltage *V* over acceleration input *Y* are defined as

$$H_{ZY}(s)=\frac{Z}{Y};\qquad H_{VY}(s)=\frac{V}{Y}$$

with the Laplace variable $s=j\omega$

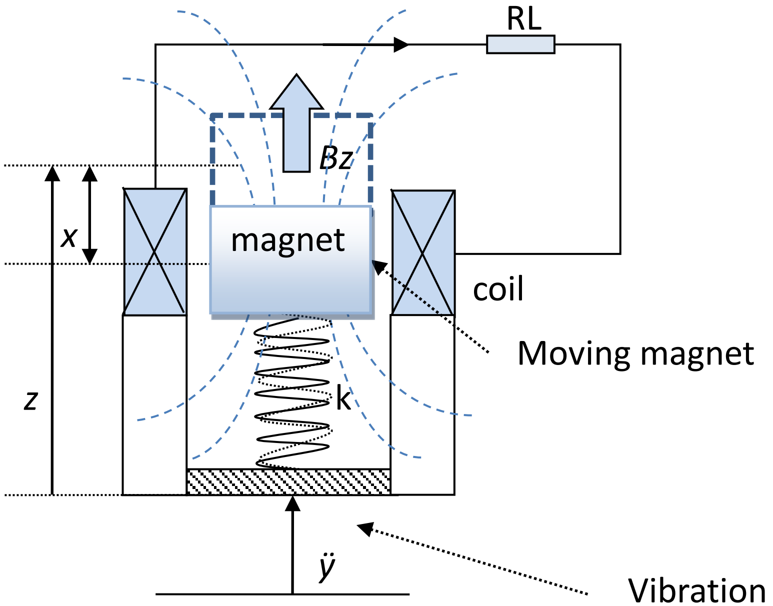
let us calculate the electrical power P_e across the resistive load R_L in frequency domain with harmonic input $\ddot{y}=Y_0e^{j\omega t}$

$$P_e(\omega)=p_e(\omega)\left|Y(j\omega)\right|^2=\frac{\left|V(j\omega)\right|^2}{2R_L}=\frac{\left|H_{VY}(j\omega)\right|^2\left|Y(j\omega)\right|^2}{2R_L}$$

$$\Rightarrow\quad\boxed{P_e(\omega)=\frac{Y_0^2}{2R_L}\left|\frac{m_2\delta_c\omega_c j\omega}{(\omega_c+j\omega)(-m_2\omega^2+d_2j\omega+k_2)+\alpha\delta\omega_c j\omega}\right|^2}$$

Electromagnetic generators

Transfer functions



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$$Z = \frac{-mY}{\det A}(s+\omega_c) = \frac{-mY(s+\omega_c)}{ms^3 + (m\omega_c + d)s^2 + (k + \alpha\delta_c\omega_c + d\omega_c)s + k\omega_c}$$

$$V = \frac{-mY}{\det A}\delta_c\omega_c s = \frac{-mY\delta_c\omega_c s}{ms^3 + (m\omega_c + d)s^2 + (k + \alpha\delta_c\omega_c + d\omega_c)s + k\omega_c}$$

the transfer functions between displacement *Z*, voltage *V* over acceleration input *Y* are defined as

$$H_{ZY}(s) = \frac{Z}{Y}; \qquad H_{VY}(s) = \frac{V}{Y}$$

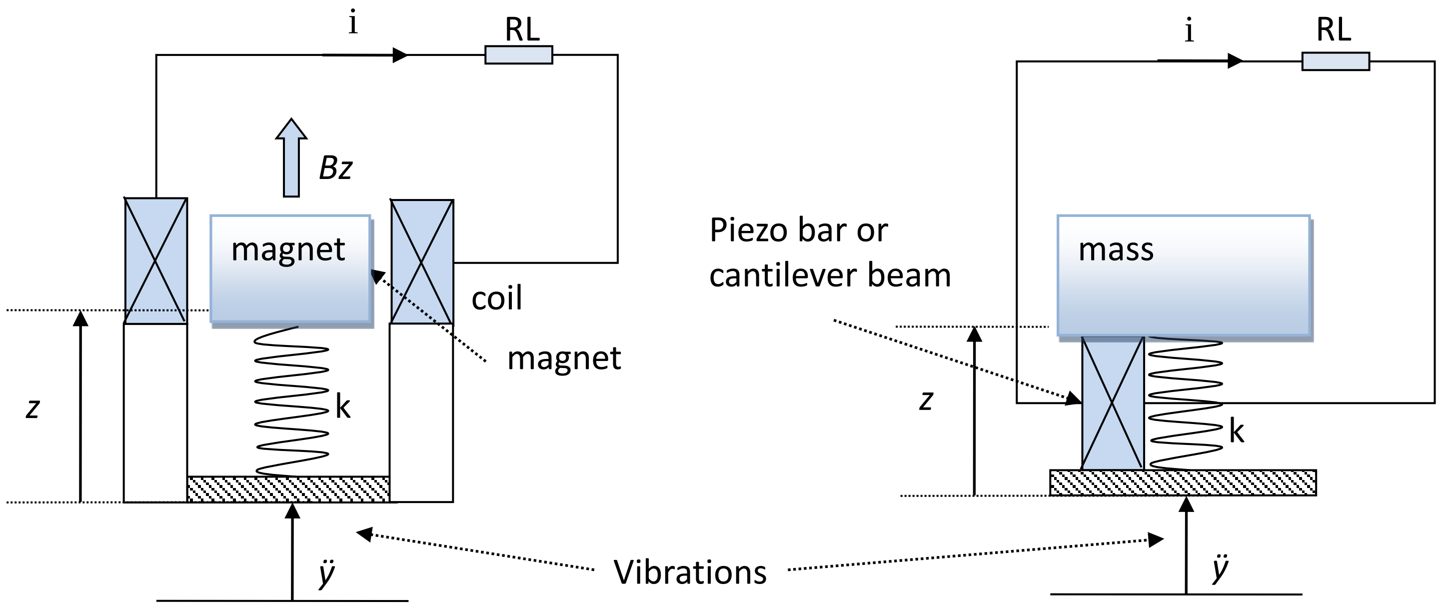
with the Laplace variable $s = j\omega$

let us calculate the electrical power P_e across the resistive load R_L in frequency domain with harmonic input $\ddot{y} = Y_0 e^{j\omega t}$

$$P_e(\omega) = p_e(\omega)\left|Y(j\omega)\right|^2 = \frac{\left|V(j\omega)\right|^2}{2R_L} = \frac{\left|H_{VY}(j\omega)\right|^2 \left|Y(j\omega)\right|^2}{2R_L} \Rightarrow \boxed{P_e(\omega) = \frac{Y_0^2}{2R_L}\left|\frac{m_2\delta_c\omega_c j\omega}{(\omega_c + j\omega)(-m_2\omega^2 + d_2 j\omega + k_2) + \alpha\delta\omega_c j\omega}\right|^2}$$

A general model

Electromagnetic transduction **Piezoelectric transduction**



Parameters	Electromagnetic	Piezoelectric	Description
α	$B_z l / R_L$	$h_{33} C_0$	Electrical restoring force factor
δ_c	$B_z l$	αR_L	Conversion coefficient
ω_c	$\dfrac{R_L}{L_e}$	$\dfrac{1}{R_L C_0}$	Characteristic cut-off frequency

$$\begin{cases} m\ddot{z} + d\dot{z} + kz = -\alpha V_L - m\ddot{y} \\ \dot{V}_L + \omega_c V_L = \delta \omega_c \dot{z} \end{cases}$$

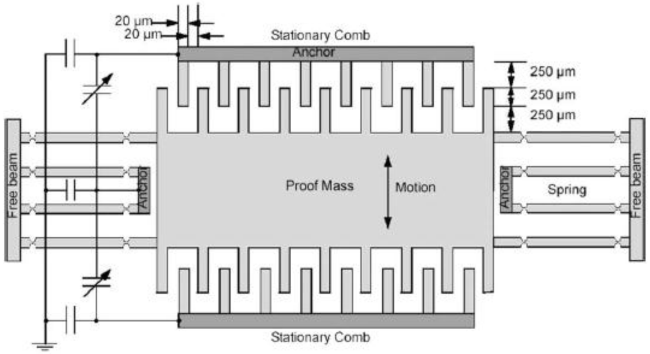
$$\ddot{y} = Y_0 e^{j\omega t}$$

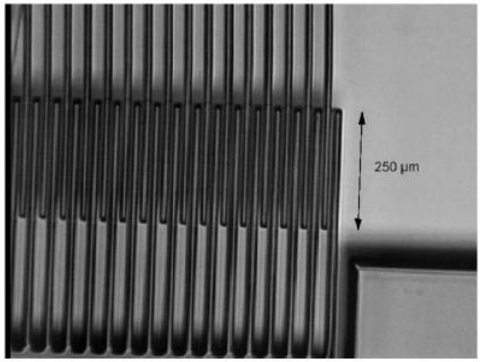
$$\boxed{P_e(\omega) = \frac{Y_0^2}{2R_L} \left| \frac{m_2 \delta_c \omega_c j\omega}{(\omega_c + j\omega)(-m_2\omega^2 + d_2 j\omega + k_2) + \alpha\delta\omega_c j\omega} \right|^2}$$

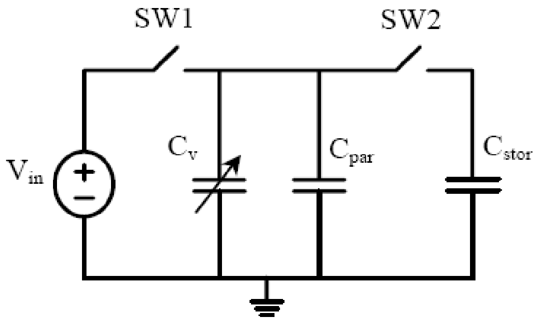
Joon Kim, K., F. Cottone, et al. (2010). "Energy scavenging for energy efficiency in networks and applications." Bell Labs Technical Journal **15**(2): 7-29.

Electrostatic generators

Operating principle (Roundy model)







Variation in capacitance causes either voltage or charge increase.

the electrostatic energy stored within capacitor is given by

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q^2 C \quad \text{with} \quad C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

for a parallel plates capacitor

At constant voltage, in order to vary the energy it's needed to counteract the electrostatic force between the mobile plates

$$F_e = \frac{1}{2}\varepsilon \frac{AV^2}{d^2}$$

while at constant charge

$$F_e = \frac{1}{2}Q\frac{2d}{\varepsilon A}$$

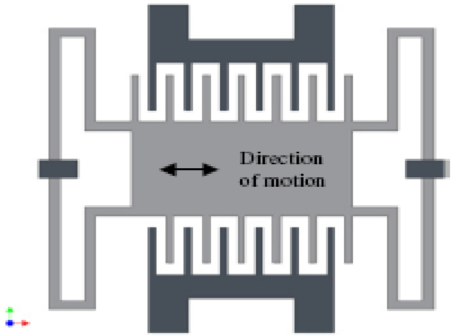
The maximum potential energy per cycle that can be harvested

$$E = \frac{1}{2}V_{in}^2 \Delta C \left(\frac{C_{\max} + C_{par}}{C_{\min} + C_{par}} \right) \Rightarrow E = \frac{1}{2}V_{\max} V_{in} \Delta C$$

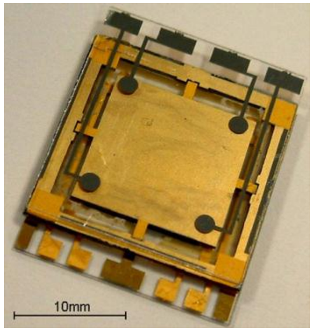
with $\Delta C = C_{max} - C_{min}$ and V_{max} which represents the maximum allowable voltage across a switch.

Electrostatic generators

In-plane



Out of plane



Operating principle

The equation of motion for the mechanical part is

$$m\ddot{x}(t)+(d_m+d_e)\dot{x}(t)+kx(t)=-m\ddot{y}(t)$$

With the system operating in sinusoidal steady state, the electrical power output is the average power converted by the electrostatic damper

$$P_e(\omega)=\frac{1}{2}d_e\omega^2X^2=\frac{1}{2}d_e\frac{m^2\omega^6Y_0^2}{(k-m\omega^2)^2+(d_e+d_m)^2\omega^2}$$

At resonance $$P_e(\omega)=\frac{1}{2}\frac{d_e}{(d_e+d_m)}m^2\omega^4Y_0^2$$

Defining the acceleration input $$A=\omega^2Y_0$$ $$P_e(\omega)=\frac{1}{2}\frac{d_e}{(d_e+d_m)}m^2A^2$$

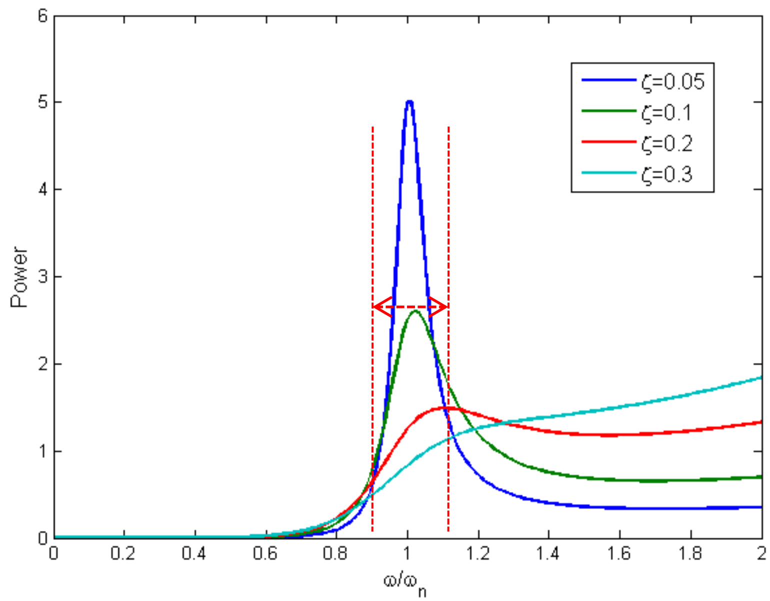
The average mechanical dissipated power $$P_e(\omega)=\frac{1}{2}d_m\omega^2X^2$$

at resonance $$P_m(\omega)=\frac{1}{2}\frac{d_m}{d_e+d_m}m^2A^2$$

Main technical limits of VEHs

Present limits of resonant systems

- narrow bandwidth that implies constrained resonant frequency-tuned applications
- small inertial mass and maximum displacement at MEMS scale
- low output voltage (~0,1V) for electromagnetic systems
- limited power density at micro scale (especially for electrostatic converters), not suitable for milliwatt electronics (10-100mW)
- versatility and adaptation to variable vibration sources
- Miniaturization issues (micromagnets, piezo beam)



At 20% off the resonance the power falls by 80-90%

Transduction techniques comparison

- Piezoelectric transducers
  - provide suitable output voltages and are well adapted for miniaturization, e.g. in MEMS applications,
  - however, the electromechanical coupling coefficients for piezoelectric thin films are relatively small, and relatively large load impedances are typically required for the piezoelectric transducer to reach it optimum working point.

- Variable capacitors
  - well suited for MEMS applications,
  - but they have relatively low power density, and they need to be charged to a reference voltage by an external electrical source such as a battery.

- Electromagnetic transducers
  - well suited for operation at relatively low frequencies in devices of medium size to drive loads of relatively low impedance,
  - they are expensive to integrate in microsystems because micro-magnets are complex to manufacture, and relatively large mass displacements are required.

Transduction techniques comparison

Type	Advantages	Disadvantages
Electromagnetic	• no need of smart material • no external voltage source	• bulky size: magnets and pick-up coil • difficult to integrate with MEMS • max voltage of 0.1V
Electrostatic	• no need of smart material • compatible with MEMS • voltages of 2~10V	• external voltage (or charge) source • mechanical constraints needed • capacitive
Piezoelectric	• no external voltage source • high voltages of 2~10V • compact configuration • compatible with MEMS • high coupling in single crystals	• depolarization • brittleness in PZT • poor coupling in piezo-film (PVDF) • charge leakage • high output impedance
Magnetostrictive	• ultra-high coupling coefficient >0.9 • no depolarization problem • high flexibility • suited to high frequency vibration	• non-linear effect • pick-up coil • may need bias magnets • difficult to integrate with MEMS

Wang, L. and F. Yuan (2007).
Energy harvesting by magnetostrictive material (MsM) for powering wireless sensors in SHM.
SPIE Smart Structures and Materials

Performance metrics

Possible definition of effectiveness $E_H = \dfrac{\text{Useful Power Output}}{\text{Maximum Possible Output}} = \dfrac{\text{Useful Power Output}}{\frac{1}{2} Y_0 Z_l \omega^3 m}.$

Power density $PD = \dfrac{El.Power}{Volume}$

Normlized power density $NPD = \dfrac{El.Power}{mass \cdot acceleration}$

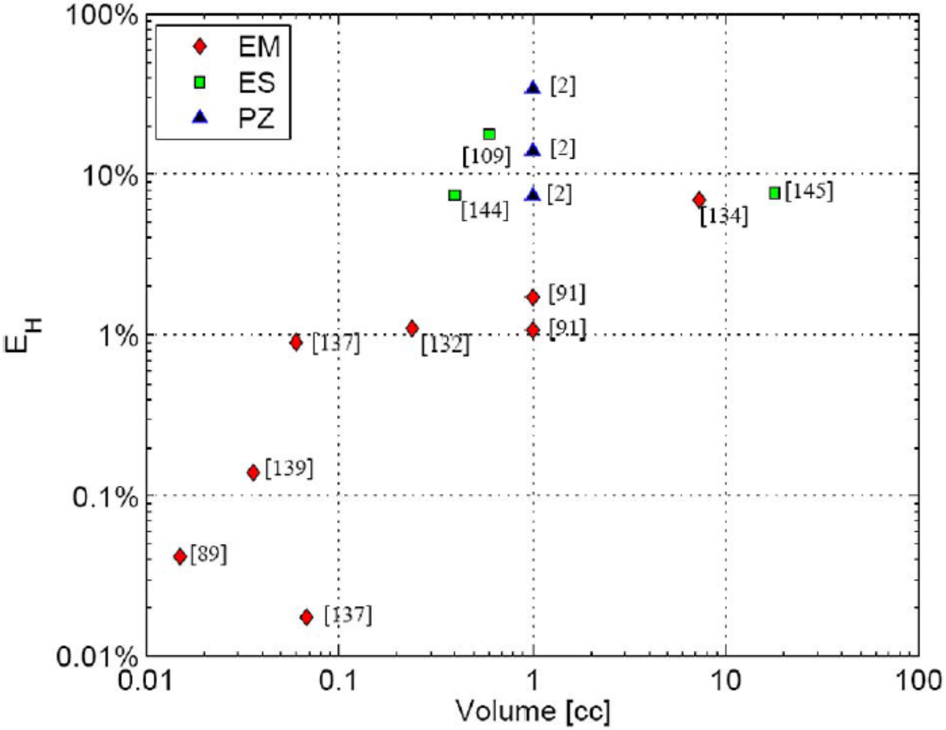
What about frequency bandwidth?

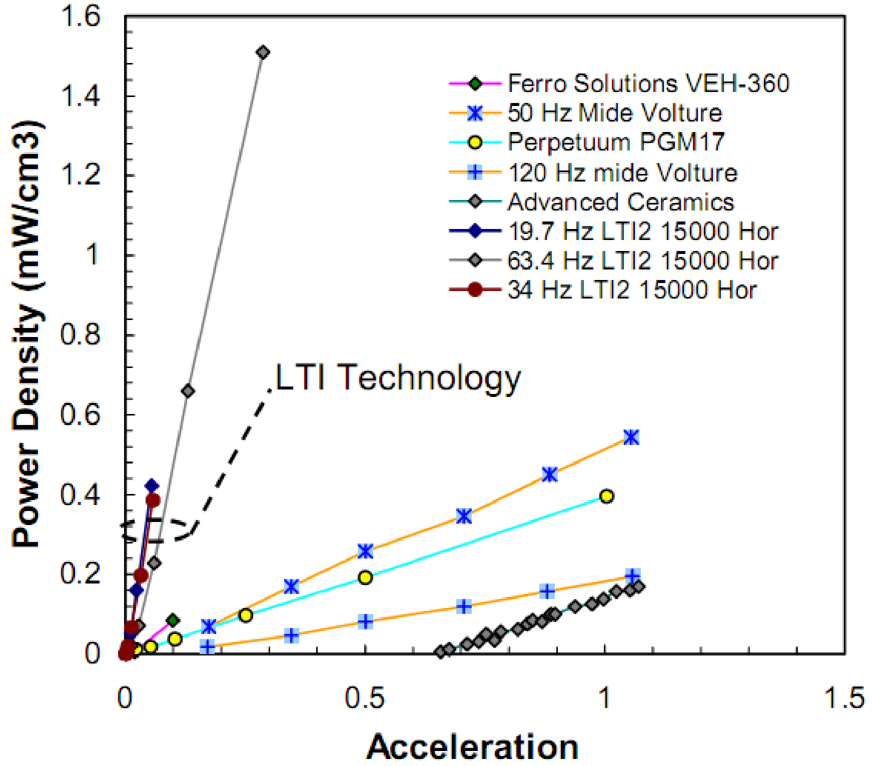
Generator[a]	Freq (Hz)	Acceln (m s^{-2})	Inertial mass (g)	Volume (cm^3)	Power (μW)	NPD (kgs m^{-3})
VIBES Mk2 EM	52	0.589	0.66	0.15	46	883.97
Glynne-Jones [13] EM	99	6.85	2.96	4.08	4990	26.07
Perpetuum [14] EM	100	0.400	50	30	4000	833.33
Ching [15] EM	110	95.5	0.192	1	830	0.09
White [16] PZ	80	2.3	0.8	0.125	2.1	3.18
Roundy [17] PZ	120	2.5	9.15	1	375	60.00
Hong [18] PZ	190	71.3	0.01	0.0012	65	10.67
Jeon [19] PZ	13 900	106.8	2.20×10^{-07}	0.000 027	1	3.25
Mitcheson [20] ES	30	50	0.1	0.75	3.7	0.002
Despesse [21] ES	50	8.8	104	1.8	1052	7.55

[a] Generators are labelled by technology: EM, electromagnetic; PZ, piezoelectric; ES, electrostatic.

Beeby, S., R. Torah, et al. (2007). "A micro electromagnetic generator for vibration energy harvesting." Journal of Micromechanics and Microengineering **17**: 1257.

Performance metrics

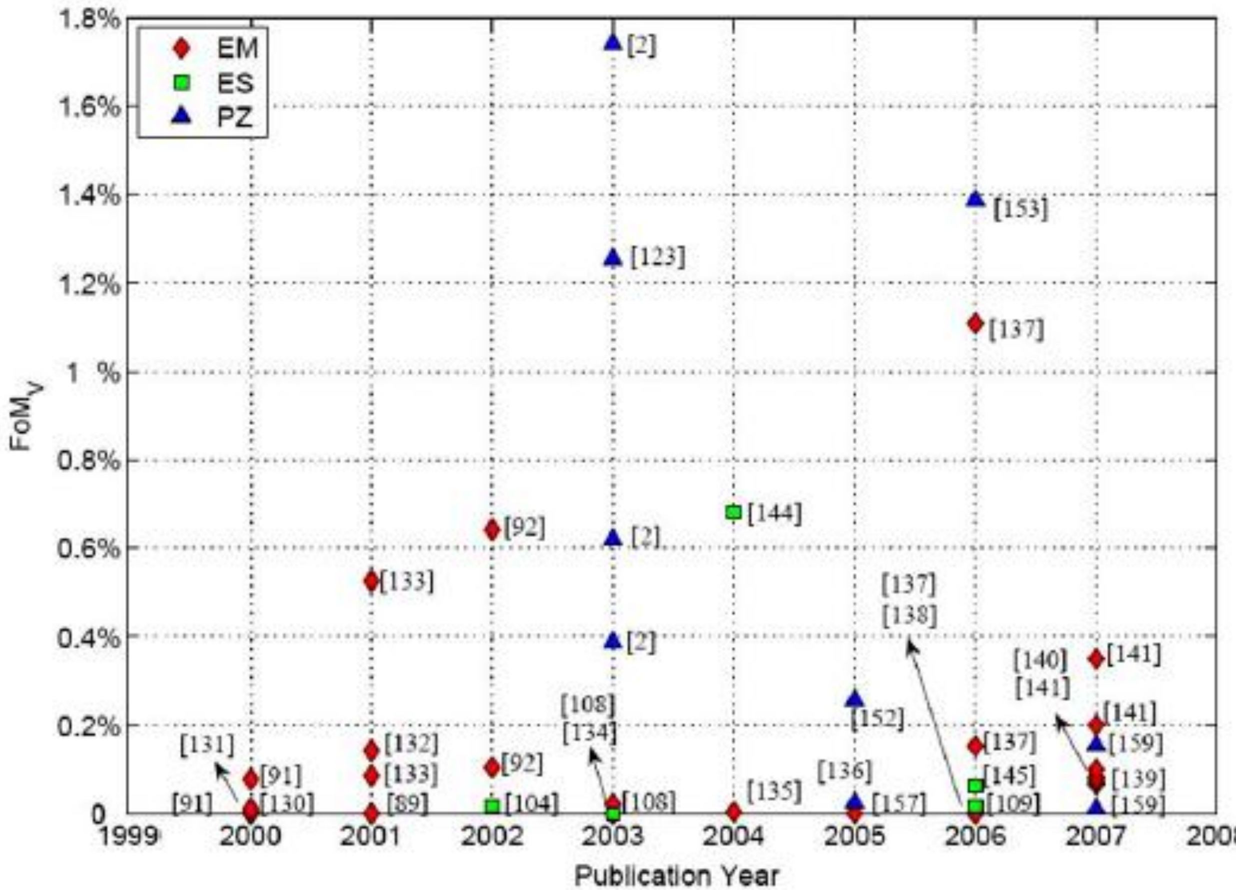




Mitcheson, P. D., E. M. Yeatman, et al. (2008). "Energy harvesting from human and machine motion for wireless electronic devices." Proceedings of the IEEE **96**(9): 1457-1486.

Performance metrics

$$\mathrm{FoM}_V = \frac{\text{Useful Power Output}}{\frac{1}{16} Y_0 \rho_{Au} Vol^{\frac{4}{3}} \omega^3}$$



Mitcheson, P. D., E. M. Yeatman, et al. (2008). "Energy harvesting from human and machine motion for wireless electronic devices." Proceedings of the IEEE **96**(9): 1457-1486.

Technical challenges and room for improvements

- Maximize the proof mass **m**
  - Improve the strain from a given mass

- Narrow frequency response and low frequency tuning
  - Actively and passive tuning resonance frequency of generator
  - Wide bandwith designs
  - Frequency up-conversion
  - Nonlinear Dynamical Systems

- Miniaturization: coupling coefficient at small scale and power density
  - Improvements of Thin-film piezoelectric-material properties
  - Improving capacitive design
  - Micro magnets implementation

- Efficient conditioning electronics
  - Integrated design
  - Power-aware operation of the powered device

Conclusions

- Vibrations represents one of the most promising renewable and reliable solutions for mobile elctronics powering.
- 90% of WSNs cannot be enabled without Energy Harvesting technologies.
- Most of vibrational energy sources are inconsistent and have relative low frequency.
- Scaling from millimeter down to micrometer size is important as well as further improvement of conversion efficiency.
- There are possible ways for efficiency improvements of Vibration Energy Harvesting technologies:
  - efficient nonlinear dynamics,
  - material properties,
  - miniaturization procedures,
  - efficient power harvesting electronics.
- A precise metrics for effectiveness is not yet well defined

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