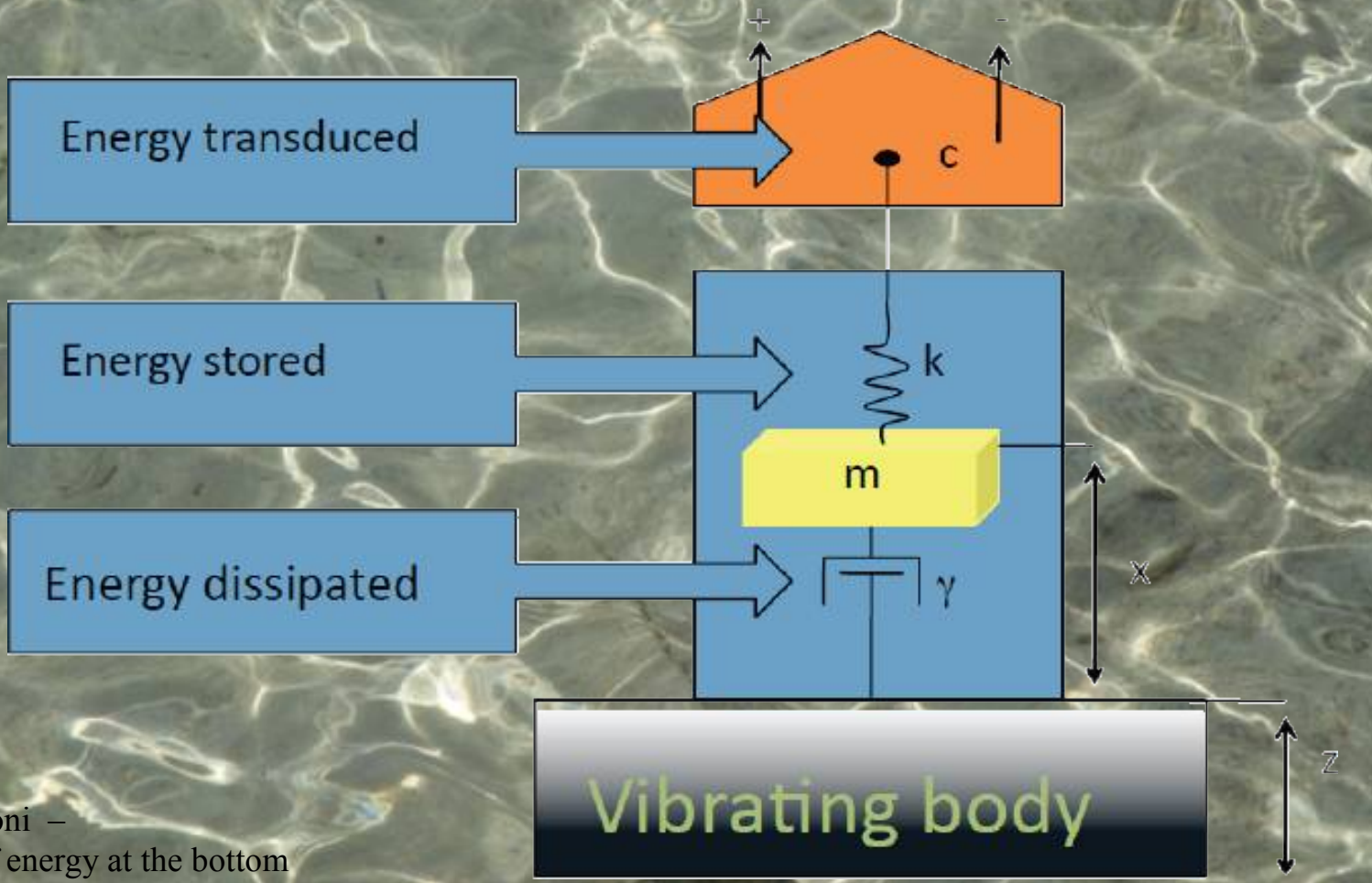
ANALYTICAL SOLUTION OF THE PIEZOELECTRIC MICRO OSCILLATOR

VALERIA NICO
UNIVERSITA' DEGLI STUDI DI PERUGIA

NiPS WORKSHOP August 2011

SCHEME OF A MICRO OSCILLATOR



From L.Gammaitoni –
"There's plenty of energy at the bottom
(Micro and nano scale nonlinear noise harvesting)"

DYNAMIC EQUATIONS

LINEAR OSCILLATOR APPROXIMATION

Potential energy

Trasduced energy

$$m\ddot{x} = -kx - \gamma\dot{x} - K_v V + \xi_z$$

Kinetics energy

Dissipated energy

Vibrating force

$$\dot{V} = K_c\dot{x} - \frac{1}{\tau}V$$

τ = RLC

TRANSFER FUNCTION

$$|H_v|^2 = \frac{\omega^2 {K_C}^2}{a^2 + b^2}$$

$$|H_x|^2 = \frac{\left(-\omega a + \frac{b}{\tau}\right)^2 + \left(\frac{a}{\tau} + \omega b\right)^2}{a^2 + b^2}$$

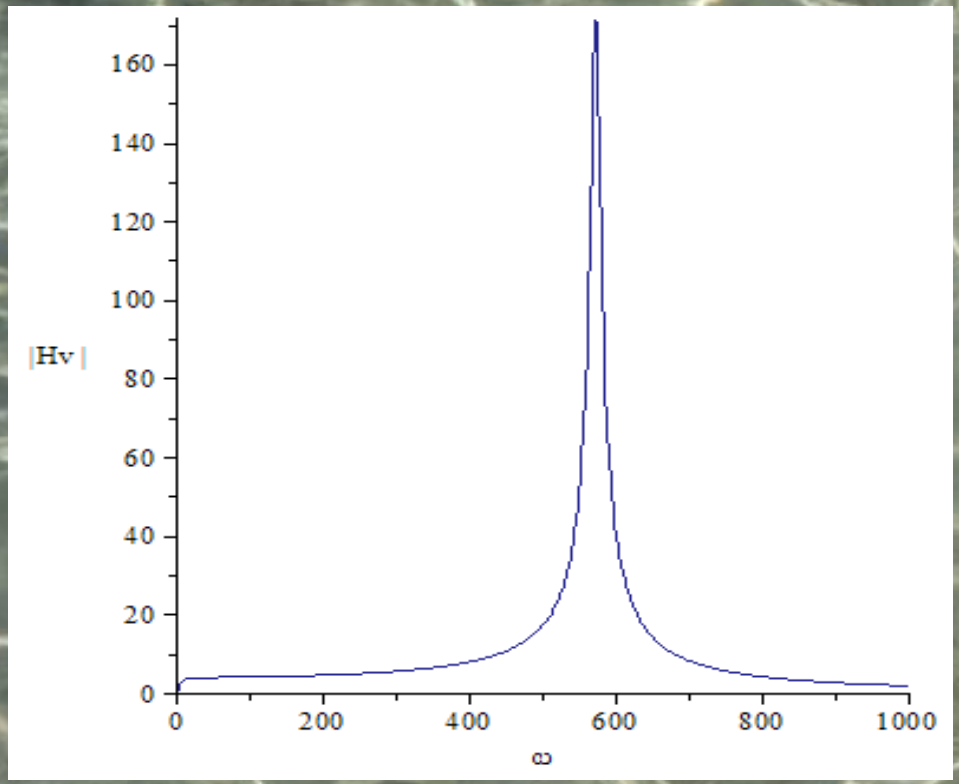
$$a = -m\omega^3 + \omega\left(k + \frac{\gamma}{\tau} + K_c K_v\right)$$

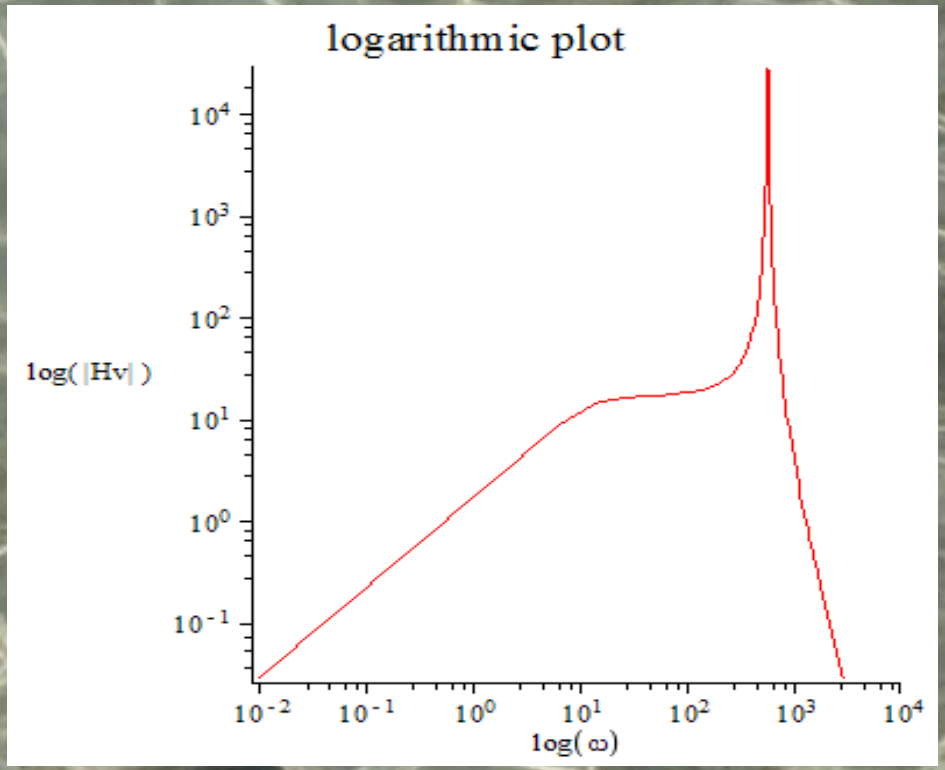
$$b = \omega^2\left(\gamma + \frac{m}{\tau}\right) - \frac{k}{\tau}$$

VALUES OF THE PARAMETRES

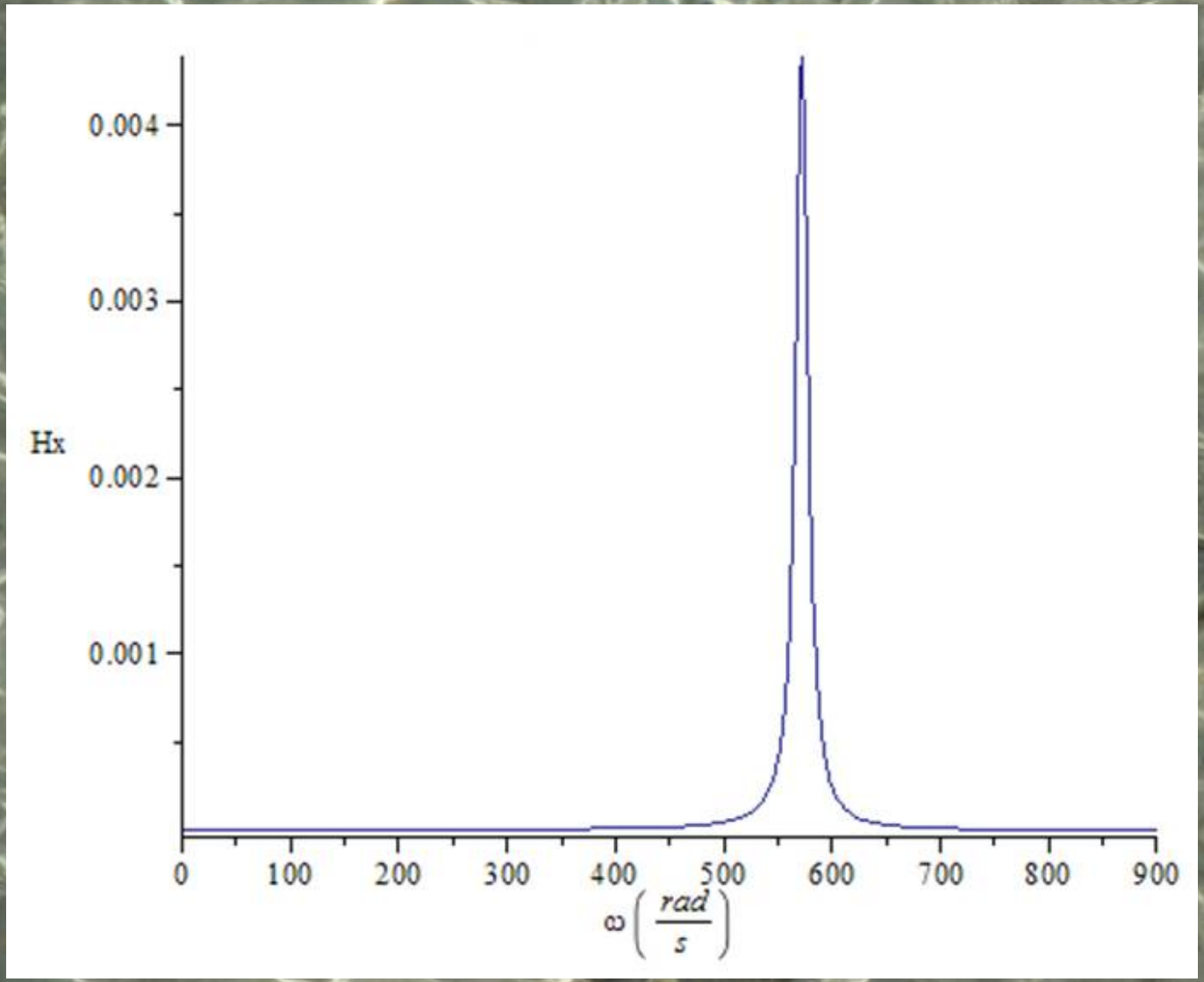
- $\tau = 0.1088\ s;$
- $Kc = 2593\ V/m;$
- $\gamma = 0.021\ kg/s;$
- $m = 0.021\ kg/s;$
- $k = 430\ N/m;$
- $Kv = 0.074\ N/V.$

VOLTAGE TRANSFER FUNCTION





DISPLACEMENT TRANSFER FUNCTION



DYNAMIC SYSTEM

$$\begin{cases} m\ddot{x} = -kx - \gamma\dot{x} - K_v V + F\sin\omega t \\ \dot{V} = K_c\dot{x} - \dfrac{1}{\tau_p}V \end{cases}$$

INITIAL CONDITIONS

- $\dot{x}(0) = 0$ m/s;
- $x(0) = 0{,}0001$ m;
- $V(0) = 1$ V.

VALUES OF THE PARAMETERS

- $\tau = 0.1088\ s;$
- $Kc = 2593\ V/m;$
- $\gamma = 0.021\ kg/s;$
- $m = 0.021\ kg/s;$
- $k = 430\ N/m;$
- $Kv = 0.074\ N/V;$
- $F=10\ N.$

SOLUTION IN TIME-SPACE

$$xtot := t \to c1\, e^{-\alpha t}\left(w12\cos(\beta t) - w22\sin(\beta t)\right) + c2\, e^{-\alpha t}\left(w12\sin(betat) + w22\cos(\beta t)\right) - c3\, e^{-lambdat}$$

$$+ \frac{1}{2\,w13\,w32\,w21 - 2\,w13\,w31\,w22 - 2\,w33\,w12\,w21 + 2\,w11\,w33\,w22}\left(e^{\alpha t}\left(w12\cos(\beta t) - w22\sin(\beta t)\right)\left(Fw33\,w12\left(-\frac{\alpha\, e^{-\alpha t}\cos\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2} - \frac{\left(\omega-\beta\right)e^{-\alpha t}\sin\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2}\right) - Fw33\,w12\left(-\frac{\alpha\, e^{-\alpha t}\cos\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2} - \frac{\left(-\omega-\beta\right)e^{-\alpha t}\sin\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2}\right) + Fw33\,w22\left(\frac{\left(-\omega-\beta\right)e^{-\alpha t}\cos\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2} - \frac{\alpha\, e^{-\alpha t}\sin\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2}\right) - Fw33\,w22\left(\frac{\left(\omega-\beta\right)e^{-\alpha t}\cos\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2} - \frac{\alpha\, e^{-\alpha t}\sin\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2}\right) - Fw32\,w13\left(-\frac{\alpha\, e^{-\alpha t}\cos\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2} - \frac{\left(\omega-\beta\right)e^{-\alpha t}\sin\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2}\right) + Fw32\,w13\left(-\frac{\alpha\, e^{-\alpha t}\cos\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2} - \frac{\left(-\omega-\beta\right)e^{-\alpha t}\sin\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2}\right)\right)\right)$$

$$+ \frac{1}{2\,w13\,w32\,w21 - 2\,w13\,w31\,w22 - 2\,w33\,w12\,w21 + 2\,w11\,w33\,w22}\left(e^{\alpha t}\left(w12\sin(\beta t) + w22\cos(\beta t)\right)\left(-Fw33\,w12\left(\frac{\left(-\omega-\beta\right)e^{-\alpha t}\cos\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2} - \frac{\alpha\, e^{-\alpha t}\sin\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2}\right) + Fw33\,w12\left(\frac{\left(\omega-\beta\right)e^{-\alpha t}\cos\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2} - \frac{\alpha\, e^{-\alpha t}\sin\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2}\right) + Fw33\,w22\left(-\frac{\alpha\, e^{-\alpha t}\cos\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2} - \frac{\left(\omega-bi\right)e^{-\alpha t}\sin\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2}\right) - Fw33\,w22\left(-\frac{\alpha\, e^{-\alpha t}\cos\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2} - \frac{\left(-\omega-\beta\right)e^{-\alpha t}\sin\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2}\right) + Fw32\,w13\left(\frac{\left(-\omega-\beta\right)e^{-\alpha t}\cos\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2} - \frac{\alpha\, e^{-\alpha t}\sin\left(\left(\omega+\beta\right)t\right)}{a^2+\left(\omega+bi\right)^2}\right) - Fw32\,w13\left(\frac{\left(\omega-\beta\right)e^{-\alpha t}\cos\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2} - \frac{\alpha\, e^{-\alpha t}\sin\left(\left(-\omega+\beta\right)t\right)}{a^2+\left(-\omega+bi\right)^2}\right)\right)\right)$$

$$- \frac{w32\, e^{\lambda t}\, w22\, w13\, F\left(-\frac{\omega\, e^{-\lambda t}\cos(\omega t)}{\lambda^2+\omega^2} - \frac{\lambda\, e^{-\lambda t}\sin(\omega t)}{\lambda^2+\omega^2}\right)}{w13\,w32\,w21 - w13\,w31\,w22 - w33\,w12\,w21 + w11\,w33\,w22}$$

SOLUTION IN TIME-SPACE

$$Vtot := t \to c1\, e^{-\alpha t}\cos(\beta t) + c2\, e^{-\alpha t}\sin(\beta t) + c3\, e^{-\lambda t} + \frac{1}{2w13w32w21 - 2w13w31w22 - 2w33w12w21 + 2w11w33w22}\left(e^{\alpha t} w13\cos(\beta t)\left(Fw33w12\left(\right.\right.\right.$$

$$-\frac{\alpha e^{-\alpha t}\cos((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2} - \frac{(\omega-\beta)e^{-\alpha t}\sin((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2}\Bigg) - Fw33w12\left(-\frac{\alpha e^{-\alpha t}\cos((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2} - \frac{(-\omega-\beta)e^{-at}\sin((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2}\right)$$

$$+Fw33w22\left(\frac{(-\omega-\beta)e^{-\alpha t}\cos((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2} - \frac{\alpha e^{-\alpha t}\sin((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2}\right) - Fw33w22\left(\frac{(\omega-\beta)e^{-\alpha t}\cos((-\omega+\beta)t)}{\alpha^2+(-\omega+bi)^2} - \frac{\alpha e^{-\alpha t}\sin((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2}\right) - Fw32w13\left(\right.$$

$$-\frac{\alpha e^{-\alpha t}\cos((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2} - \frac{(\omega-\beta)e^{-\alpha t}\sin((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2}\Bigg) + Fw32w13\left(-\frac{\alpha e^{-\alpha t}\cos((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2} - \frac{(-\omega-\beta)e^{-\alpha t}\sin((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2}\right)\Bigg)\Bigg)$$

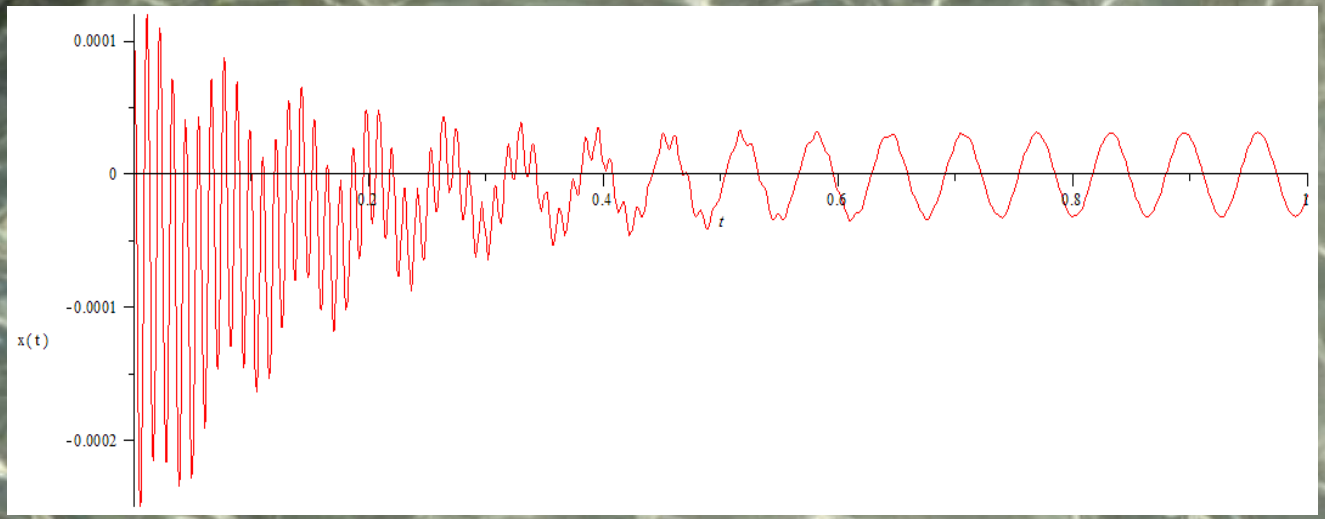
$$+\frac{1}{2w13w32w21 - 2w13w31w22 - 2w33w12w21 + 2w11w33w22}\left(e^{\alpha t} w13\sin(\beta t)\left(-Fw33w12\left(\frac{(-\omega-\beta)e^{-\alpha t}\cos((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2} - \frac{\alpha e^{-\alpha t}\sin((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2}\right)\right.\right.$$

$$+Fw33w12\left(\frac{(\omega-\beta)e^{-\alpha t}\cos((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2} - \frac{\alpha e^{-\alpha t}\sin((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2}\right) + Fw33w22\left(-\frac{\alpha e^{-\alpha t}\cos((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2} - \frac{(\omega-\beta)e^{-\alpha t}\sin((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2}\right)$$

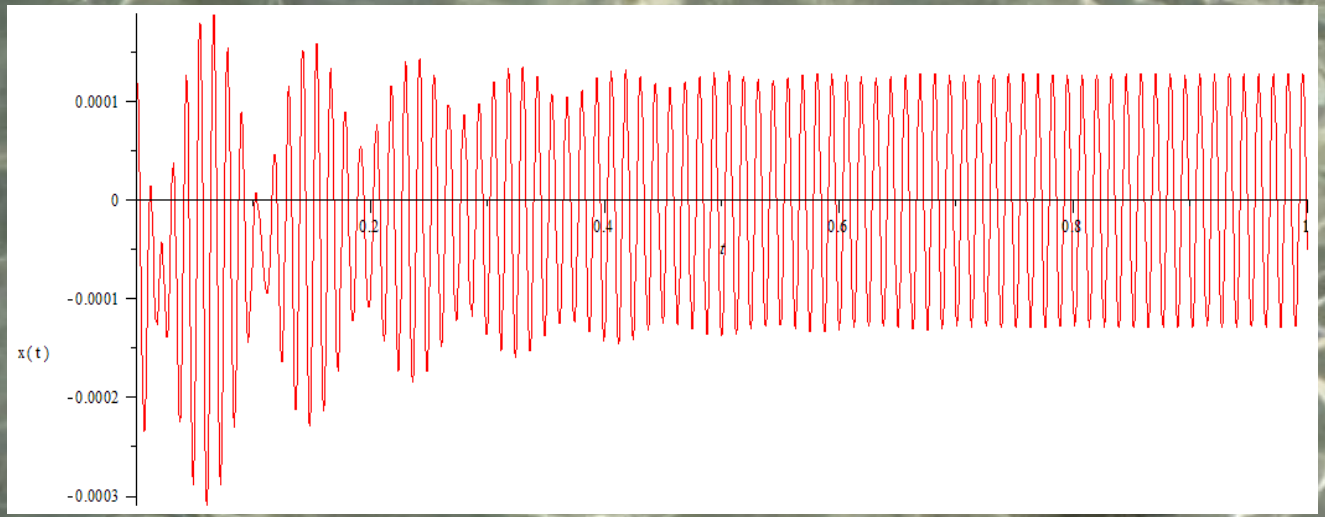
$$-Fw33w22\left(-\frac{\alpha e^{-\alpha t}\cos((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2} - \frac{(-\omega-\beta)e^{-\alpha t}\sin((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2}\right) + Fw32w13\left(\frac{(-\omega-\beta)e^{-\alpha t}\cos((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2} - \frac{\alpha e^{-\alpha t}\sin((\omega+\beta)t)}{\alpha^2+(\omega+\beta)^2}\right)$$

$$\left.\left.-Fw32w13\left(\frac{(\omega-\beta)e^{-\alpha t}\cos((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2} - \frac{\alpha e^{-\alpha t}\sin((-\omega+\beta)t)}{\alpha^2+(-\omega+\beta)^2}\right)\right)\right) - \frac{w33\, e^{\lambda t} w22w13F\left(-\frac{\omega e^{-\lambda t}\cos(\omega t)}{\lambda^2+\omega^2} - \frac{\lambda e^{-\lambda t}\sin(\omega t)}{\lambda^2+\omega^2}\right)}{w13w32w21 - w13w31w22 - w33w12w21 + w11w33w22}$$

DISPLACEMENT

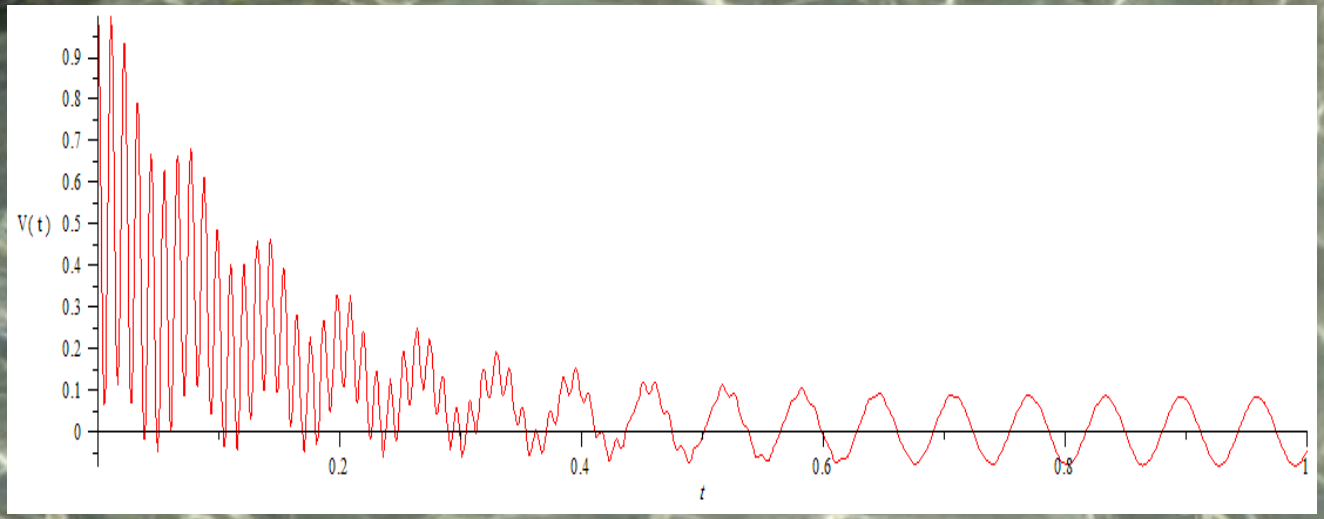


ω=100rad/s

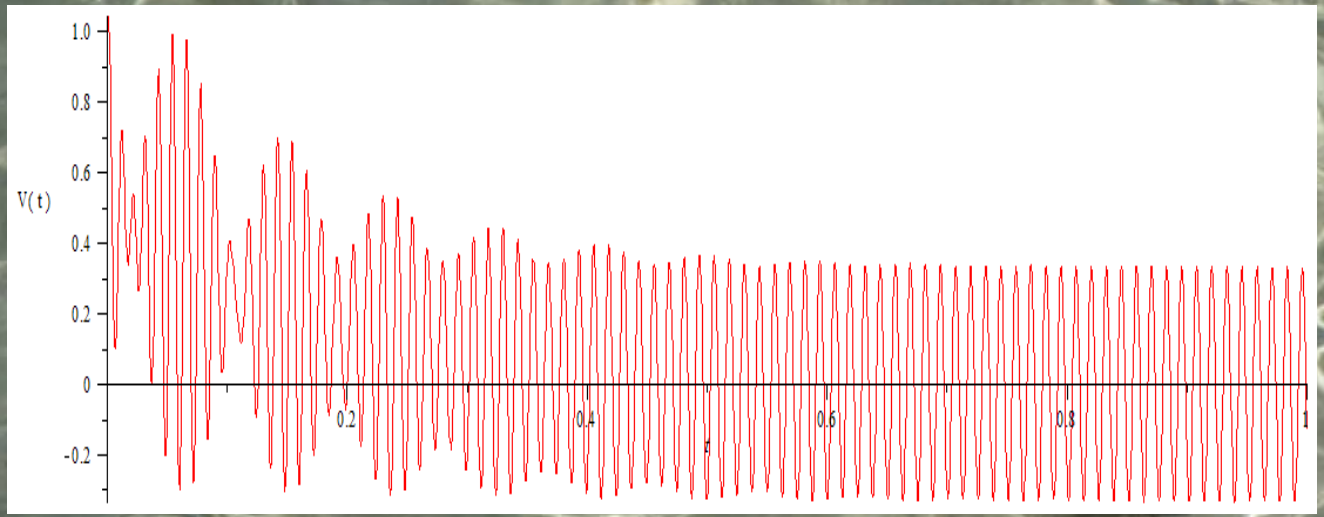


ω=500rad/s

VOLTAGE



ω=100rad/s



ω=500rad/s

Valeria Nico NiPS workshop Augoust 2011