DDAYSLAC2018
29 November, Punta del Este

The cost of remembering

(Fundamental thermodynamic limits in the physics of memories)

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NiPS Laboratory
Noise in Physical Systems

University of Perugia (IT)
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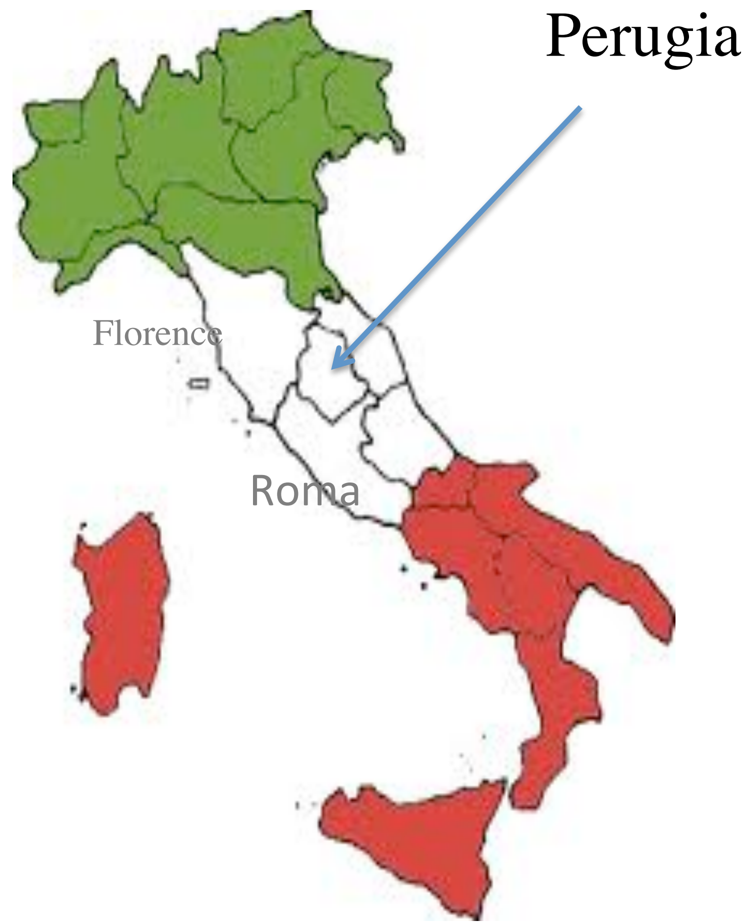


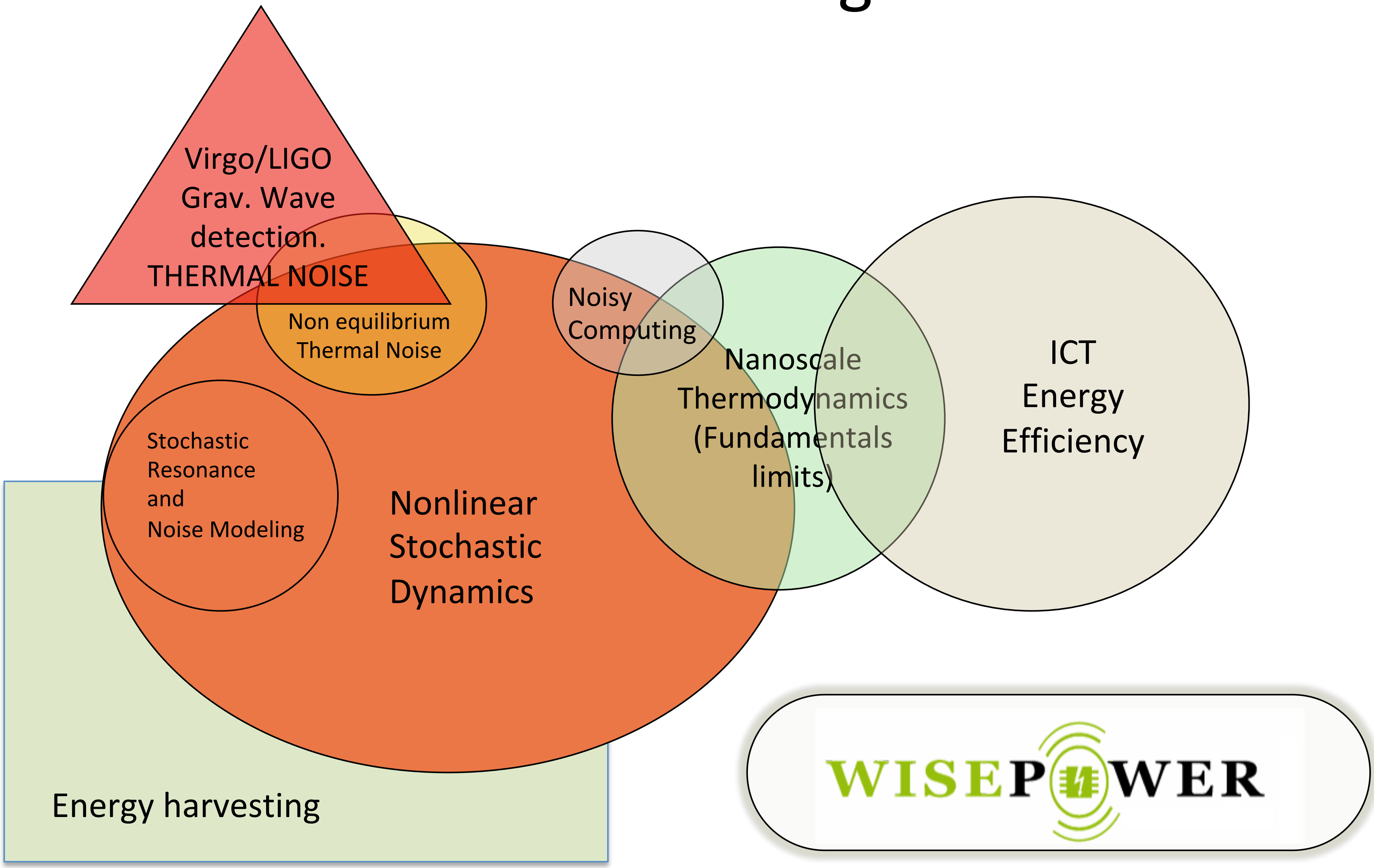
NiPS Laboratory
Noise in Physical Systems

Cristina Diamantini, Helios Vocca, Francesco Cottone, Igor Neri, Francesco Orfei, Alessandro di Michele, Flavio Travasso, Maurizio Mattarelli, Alessio Stollo, Valbona Ramci

www.nipslab.org

ICT-Energy ZEROPOWER

What are we doing at NiPS?



April 4, 2005

March 13, 2013

NiPS Laboratory
Noise in Physical Systems

We are interested in noise and fluctuations.
Energy transformation processes at micro and nano scales.

ICT-Energy
Fundamental limits in the physics of computing

Questions like:

- Can we operate a computer by spending 0 energy?
- How long can a memory last?
- How much energy dos it take to remember something?

FET Proactive projects

2006-2009 EC (SUBTLE VIFP)
2010-2013 EC (NANOPOWER VIIFP)
2010-2013 EC (ZEROPOWER VIIFP)
2012-2015 EC (LANDAUER VIIFP)
2013-2016 EC (ICT-Energy VIIFP)
2015-2018 EC (Proteus H2020)
2017-2019 EC (OPRECOMP H2020)
2017-2019 EC (ENABLES H2020)

1992-2018 Virgo-LIGO

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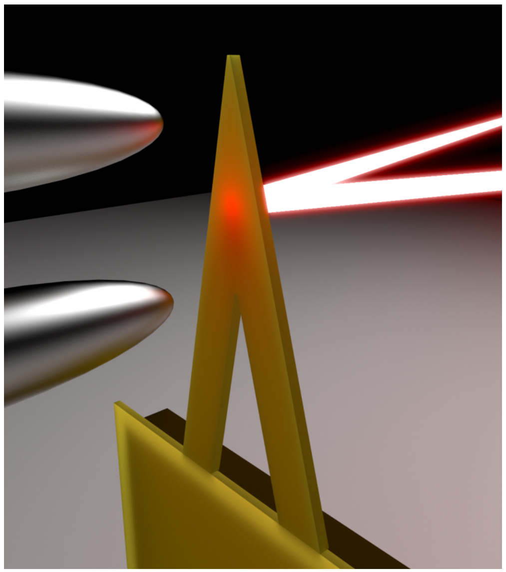
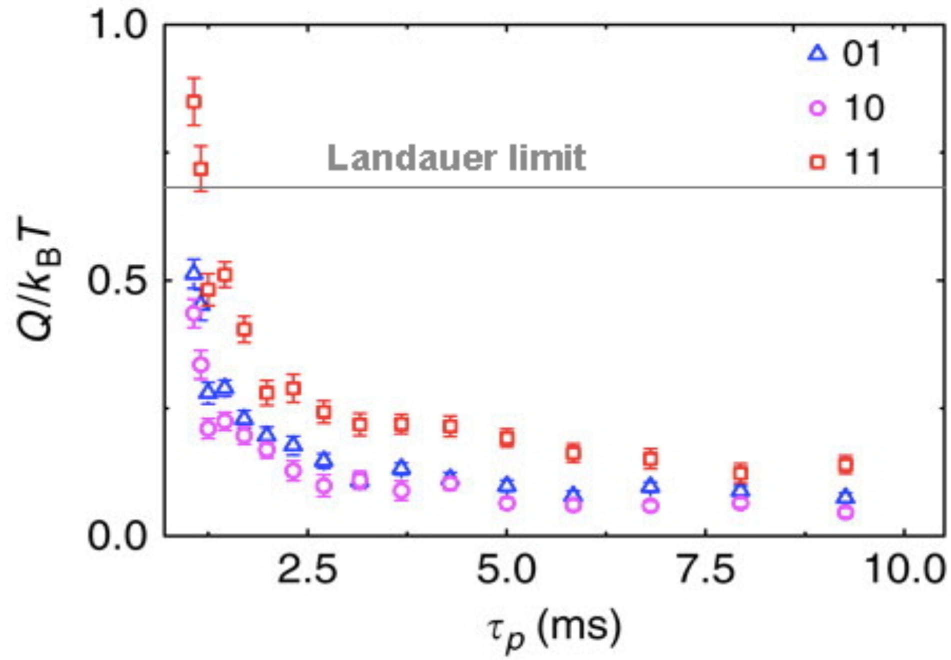
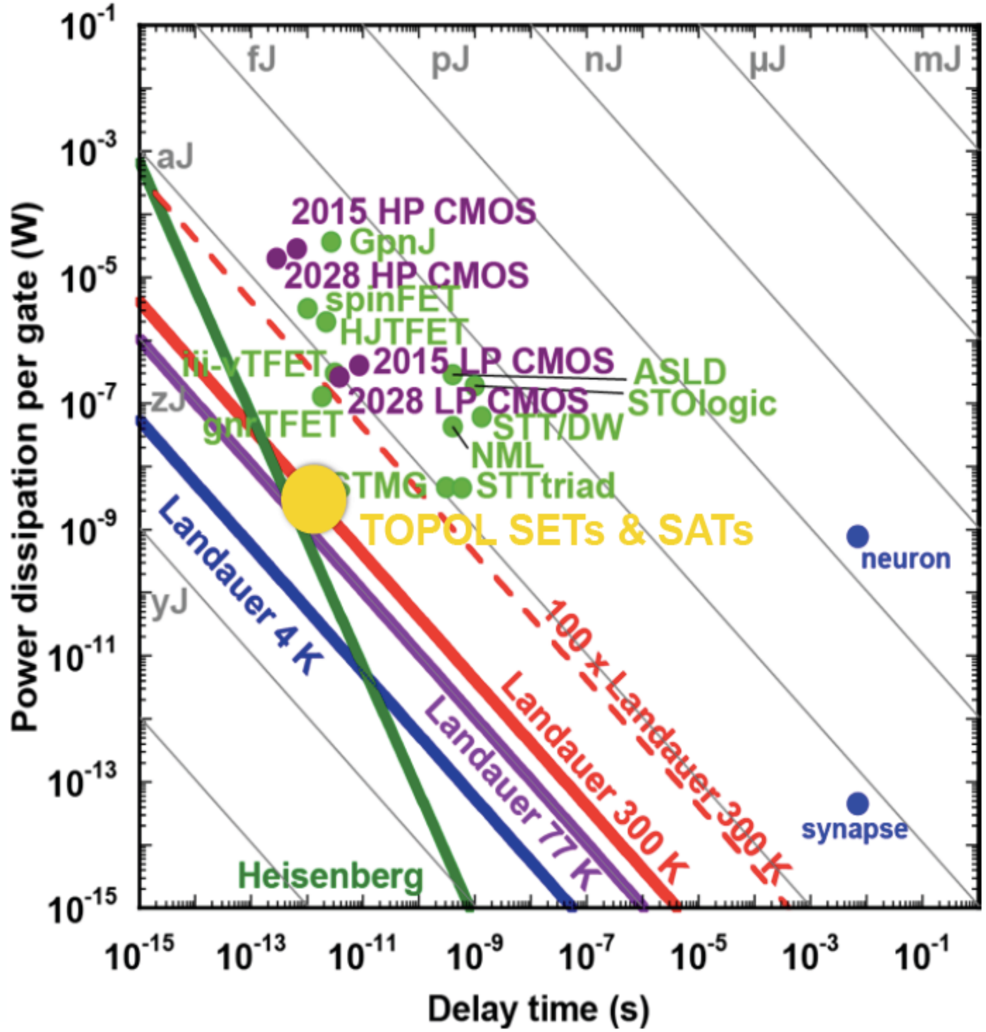
Some recent results





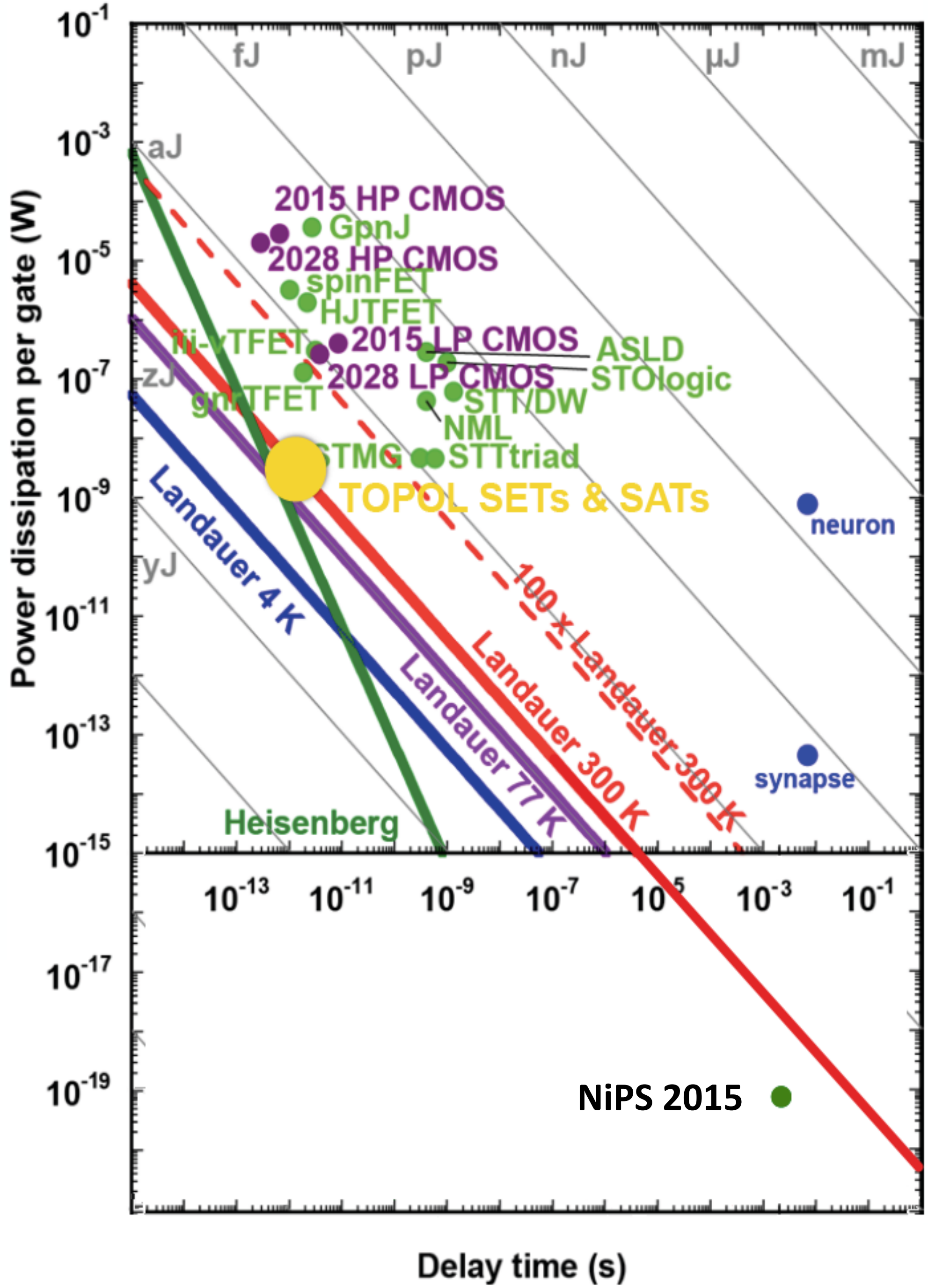
Sub-kBT micro-electromechanical irreversible logic gate,
M. López-Suárez, I. Neri, L. Gammaitoni.
Nature Communications 7, 12068 (2016)

The state of the art in energy dissipation during computation



Source: D. Paul, ICT-Energy Research Agenda, 2015

The state of the art in energy dissipation during computation

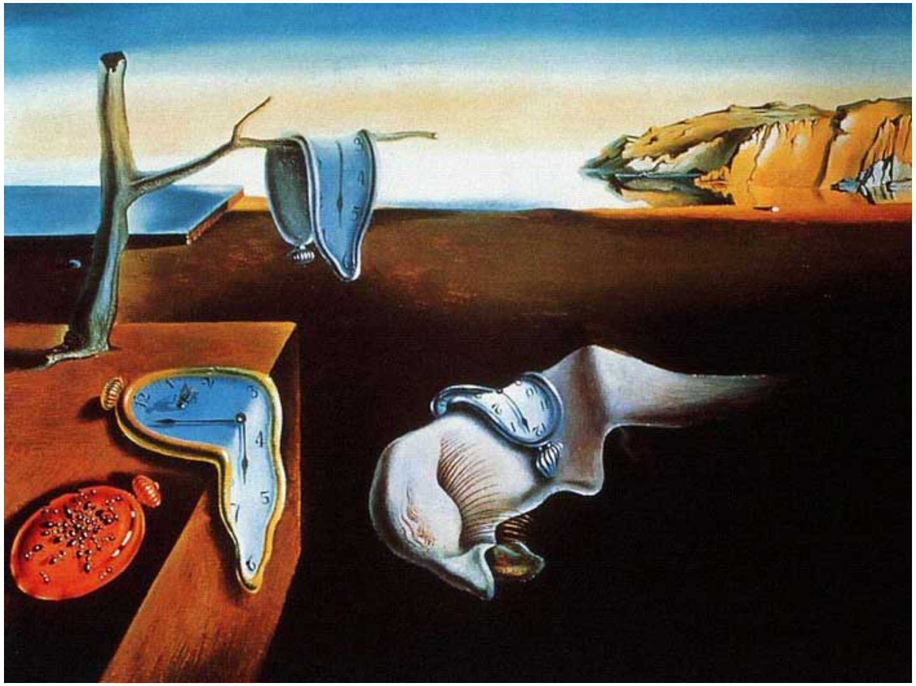


Source: D. Paul, ICT-Energy Research Agenda, 2015

Noise in Physical Systems

The cost of remembering

(Fundamental thermodynamic limits in the physics of memories)

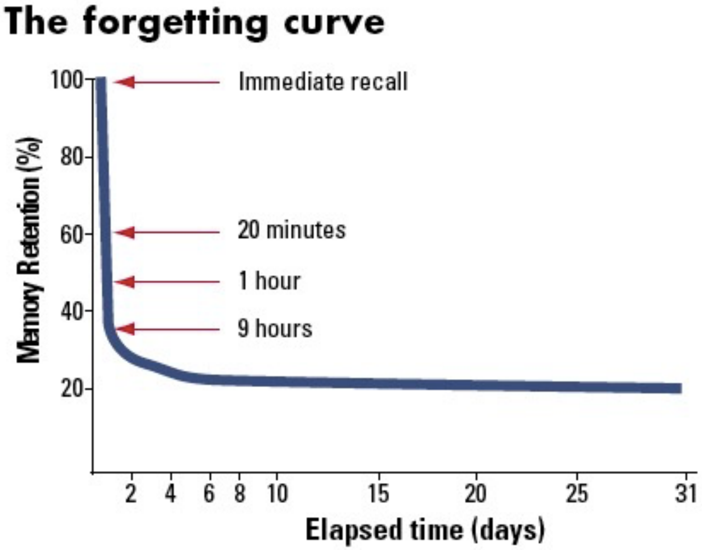


1931, The persistence of memory, S. Dalì

The act of remembering is of fundamental importance in human experience

Hermann Ebbinghaus

1885
Memory: A Contribution to Experimental Psychology



The "forgetting curve" was developed by Hermann Ebbinghaus in 1885. Ebbinghaus memorized a series of nonsense syllables and then tested his memory of them at various periods ranging from 20 minutes to 31 days. This simple but landmark research project was the first to demonstrate that there is an exponential loss of memory unless information is reinforced.

Stahl SM, Davis RL, Kim D, et al. *CNS Spectr*. Vol 15, No 8. 2010.

The memory content tends to deteriorate with time if no action is taken.
The deterioration process is approximately exponential.

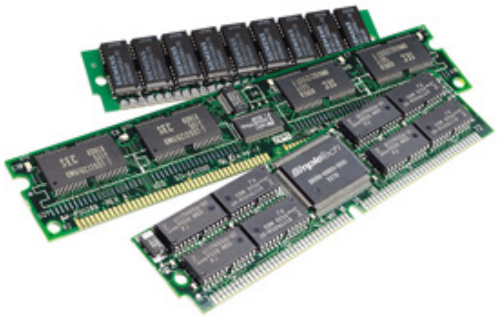
Memory deterioration occurs also in materials and artifacts that tend to loose their original shape

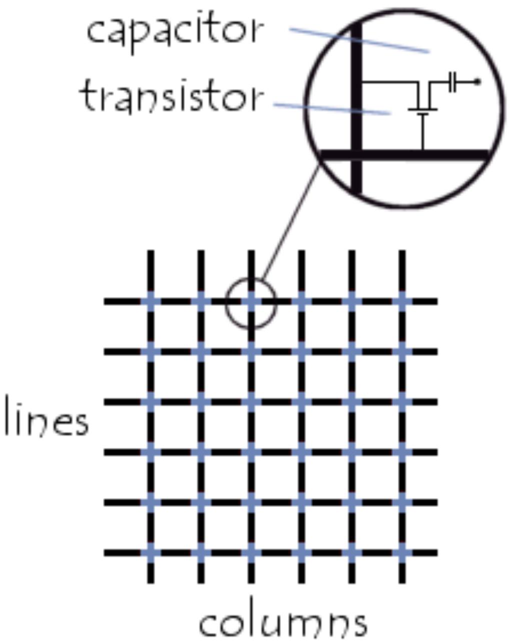
1908 1968

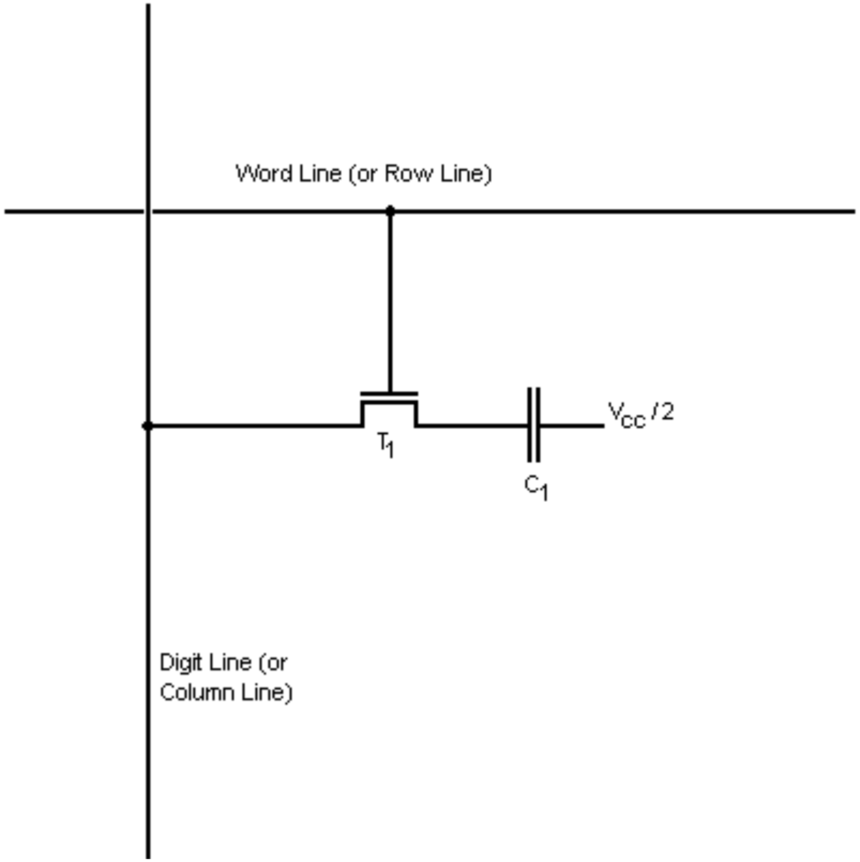
Three pictures, 10 years apart

In order to preserve the original shape, i.e. in order to keep the memory, we usually perform restoration work / memory reinforcement.

Memory degradation is a problem common also to computer memories that tend to lose their content over time

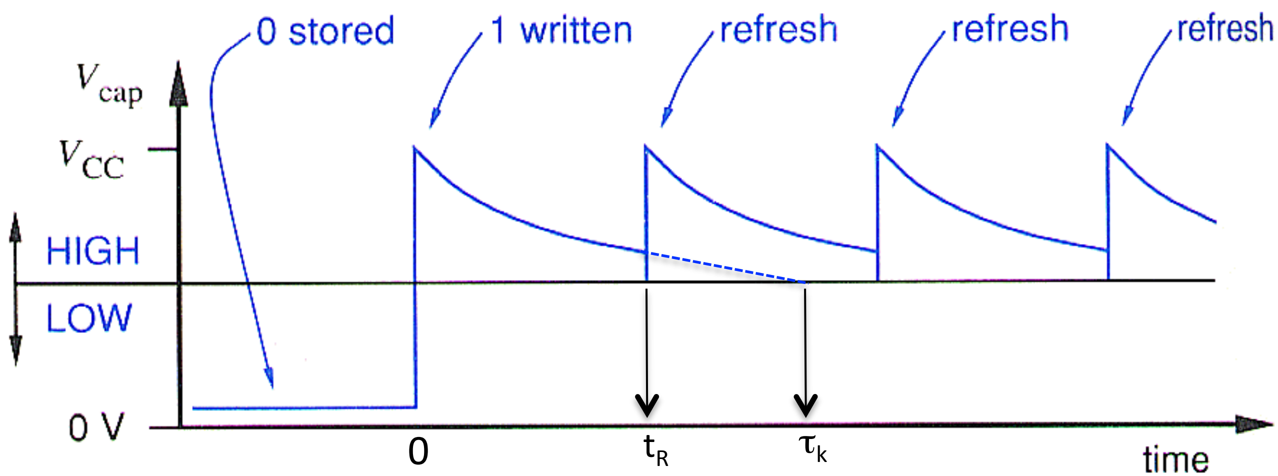






DRAM

In order to counterbalance the memory degradation, a periodic refresh operation is performed



If no refresh operation is performed the memory is lost on average after a time τ_K

The refresh operation is performed periodically with period t_R

The refresh operation last for a time t_p

$$t_p << t_R << \tau_K$$

Nano seconds

Micro seconds

seconds

Scope of the work

Assumed that the refresh operation has an energetic cost **Q**

we are interested in the **fundamental energy limits** to preserve a given bit for a time **t**

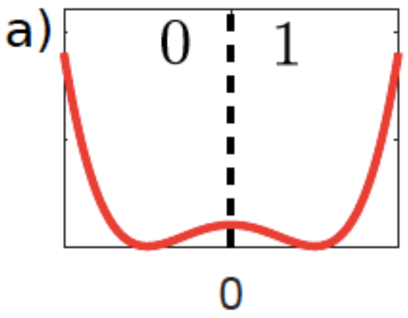
with a probability of failure not larger than **P_E**

while executing the refresh procedure with periodicity **t_R**

Plan of the work

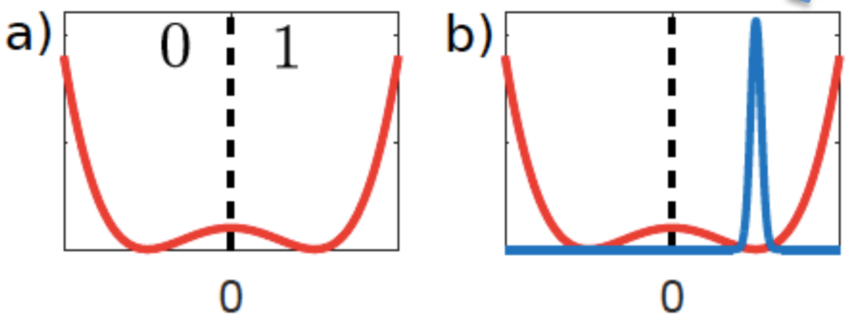
1. introduce a simple physical model for the 1-bit memory
2. Compute $\mathbf{t_R}$ for a given set of $\mathbf{P_E}$ and $\mathbf{t}$
3. Perform an experiment to determine the minimum energy required
4. Elaborate considerations

1 1-bit memory



1 1-bit memory

Initial probability density p(x,t)



1 1-bit memory

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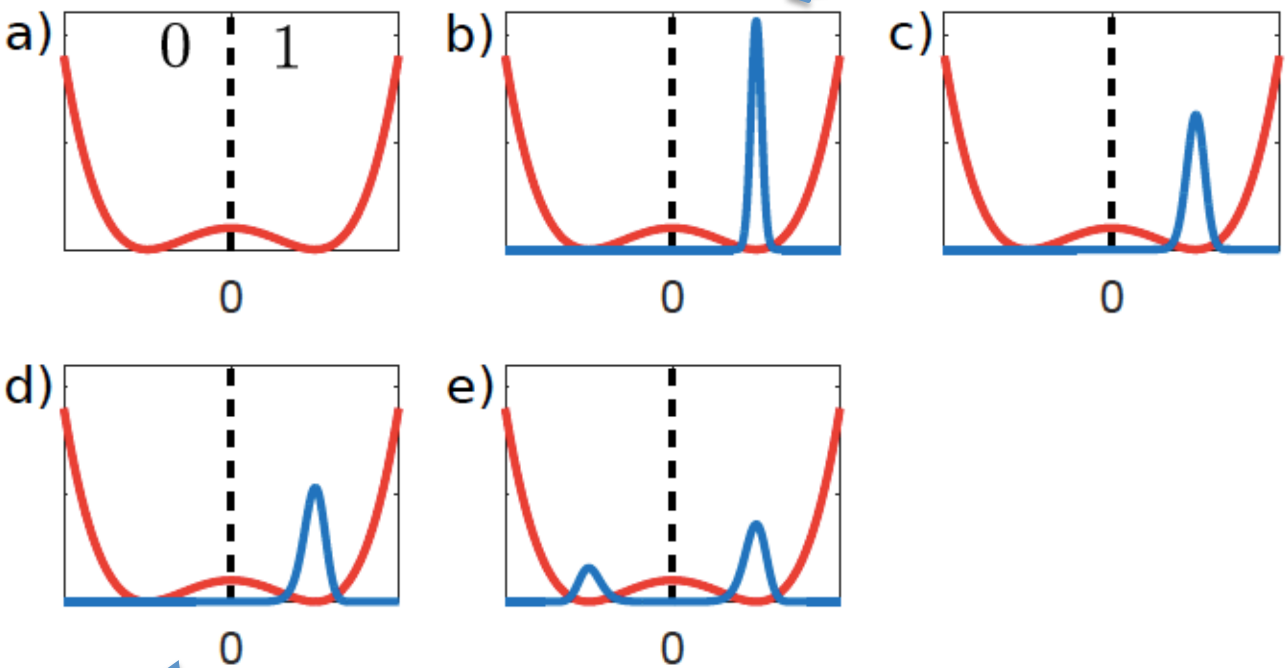
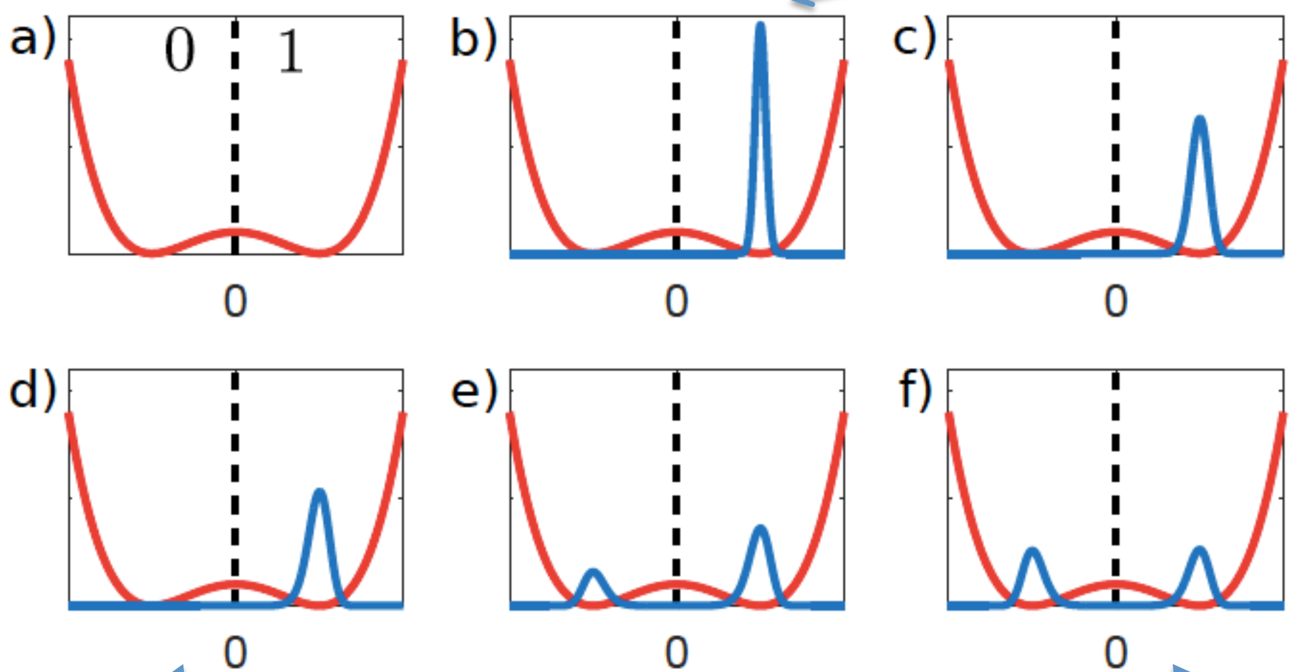


Relaxation within one well
With time scale t_w

Relaxation toward global
Equilibrium with time scale τ_K

1 1-bit memory

Initial probability density p(x,t)



Relaxation within one well
With time scale t_w

Relaxation toward global
Equilibrium with time scale τ_K

Memory lost

2 Compute $\mathbf{t_R}$ for a given set of $\mathbf{P_E}$ and $\mathbf{t}$

In this framework if we indicate with $P_0(t) = \int_{-\infty}^{0} p(x,t)\mathrm{d}x$

the probability to be in the wrong well (bit 0 instead of bit 1), we have:

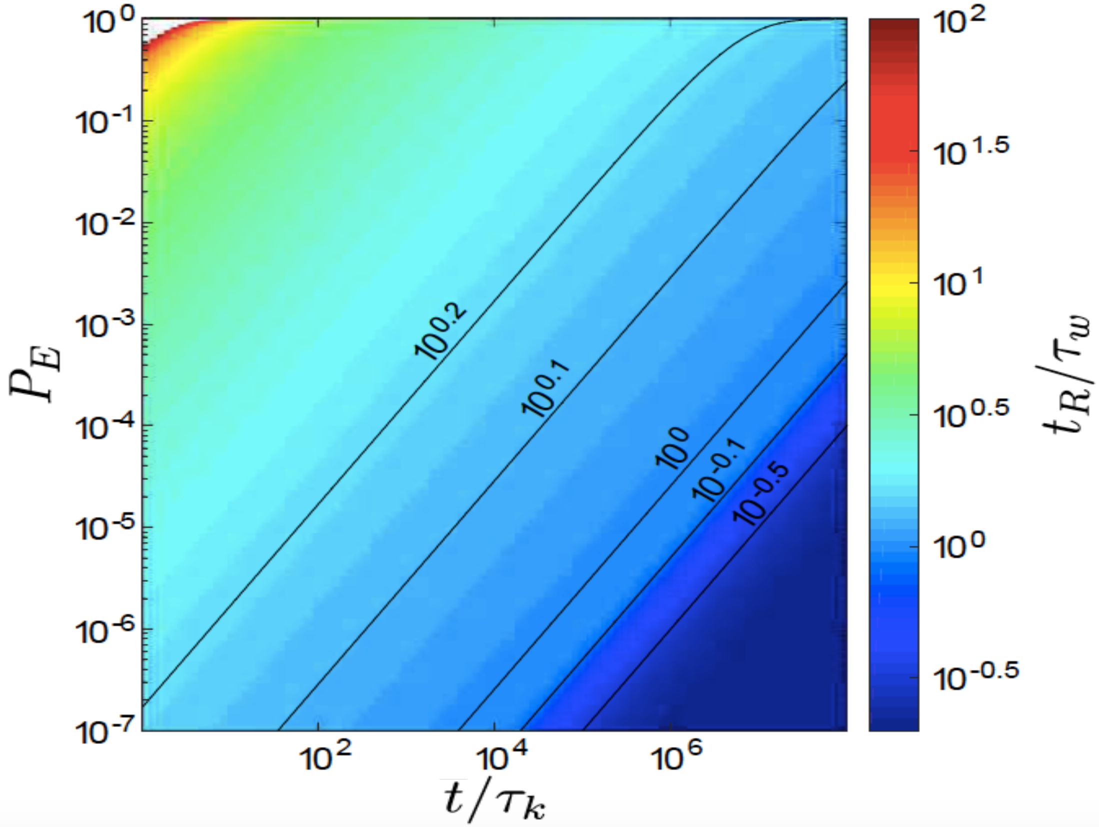
$$P_E = 1 - \left[1 - P_0\left(t_R\right)\right]^{\frac{t}{t_R}}$$

After $N = t/t_R$ refresh cycles

In order to compute this quantity we assume a bistable Duffing potential U(x). The density function p(x, t) is described via the dimensionless Fokker-Plank equation

$$\frac{\partial}{\partial t}\, p(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}\, p(x,t)\right) + \frac{k_B T}{\Delta U}\frac{\partial^2}{\partial x^2}\, p(x,t)$$

2 Compute t_R for a given set of P_E and t



For a given total duration t, the smaller is the acceptable P_E, the more frequent I have to refresh.

3 Determine the minimum energy for keeping the memory

We now consider the energy cost of a single refresh operation.
Based on our model, the refresh operation consists in bringing the p(x,t) back to its initial condition:

$$p(x,t_R) \rightarrow p(x,0)$$

We assume that the motion inside one well can be approximated by the harmonic oscillator dynamics. This is reasonable while $t_R << \tau_K$.

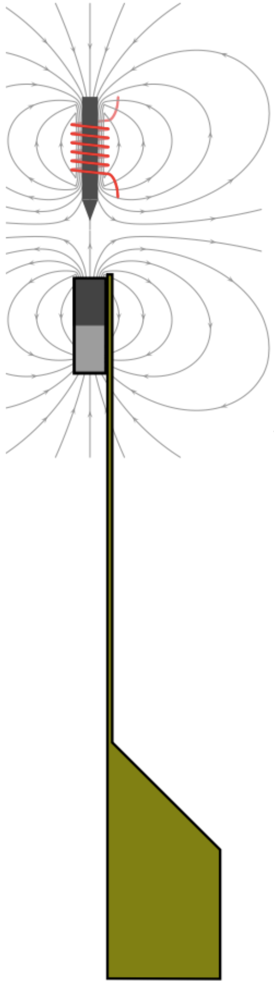
The resulting probability density function is a sum of two Gaussian peaks centred around the minima of **U**(**x**), each one with the same standard deviation σ

The refresh operation amounts to change $\sigma_f = \sigma(t_R)$ into $\sigma_i = \sigma(0)$

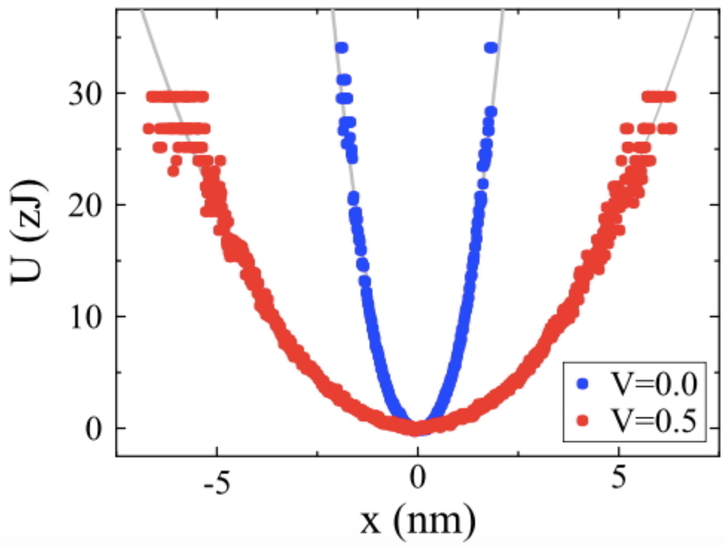
with

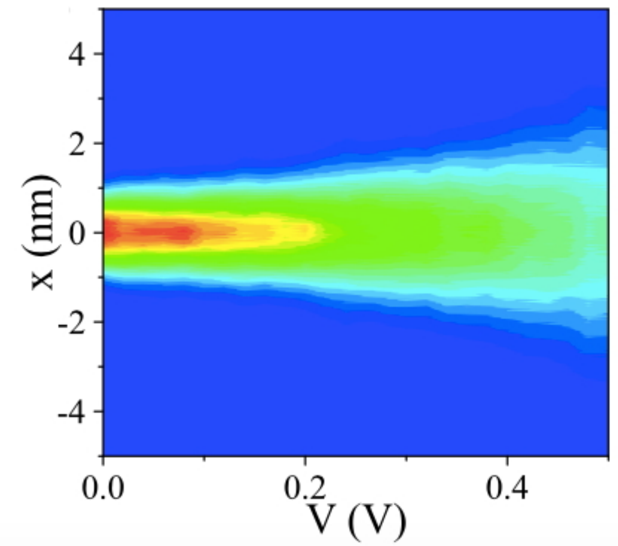
$$\sigma(t_R) = \sqrt{\sigma_w^2 + \exp\left(-\frac{t_R}{\tau_w}\right)(\sigma_i^2 - \sigma_w^2)}$$

3 Determine the minimum energy for keeping the memory

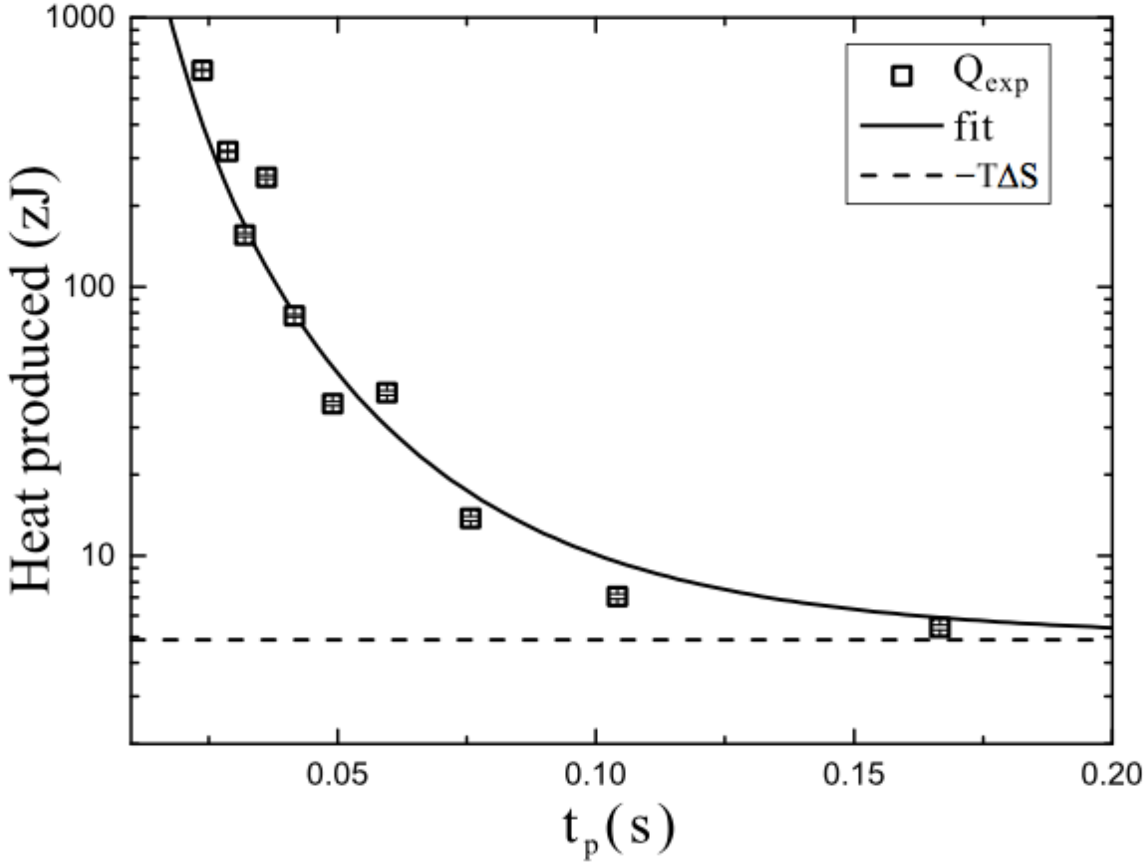


To measure the minimum energy required for the refresh, i.e. to "squeeze" the density function inside an harmonic well, we perform an experiment with a micro-mechanical V-shaped cantilever where the relevant observable x is the tip position.





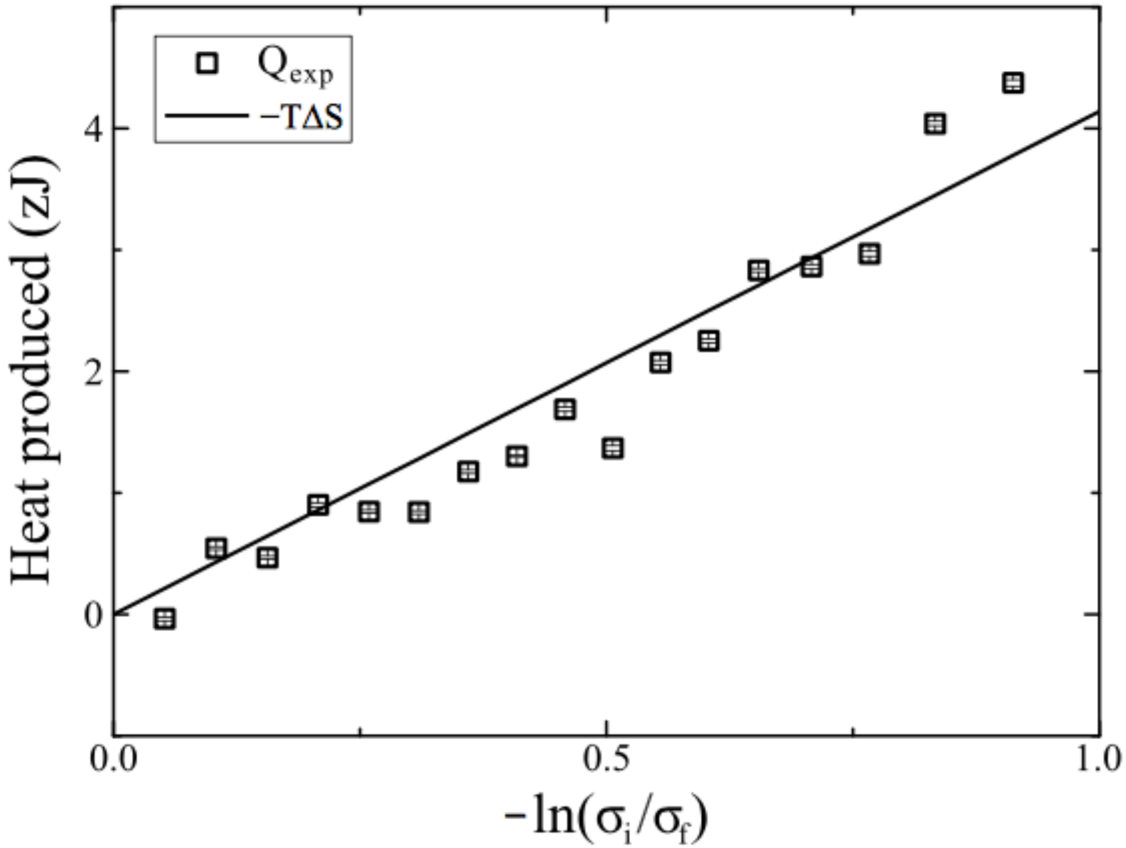
3 Determine the minimum energy for keeping the memory



$T\,\Delta S$

1 zJ = 10^{-21}J

3 Determine the minimum energy for keeping the memory



With $\Delta S = k_B \ln\left(\frac{\sigma_i}{\sigma_f}\right)$

NiPS Laboratory Noise in Physical Systems

3 Determine the minimum energy for keeping the memory

Once we understood that the theoretical limit can be reached we write its expression for the Duffing potential within the assumed approximation:

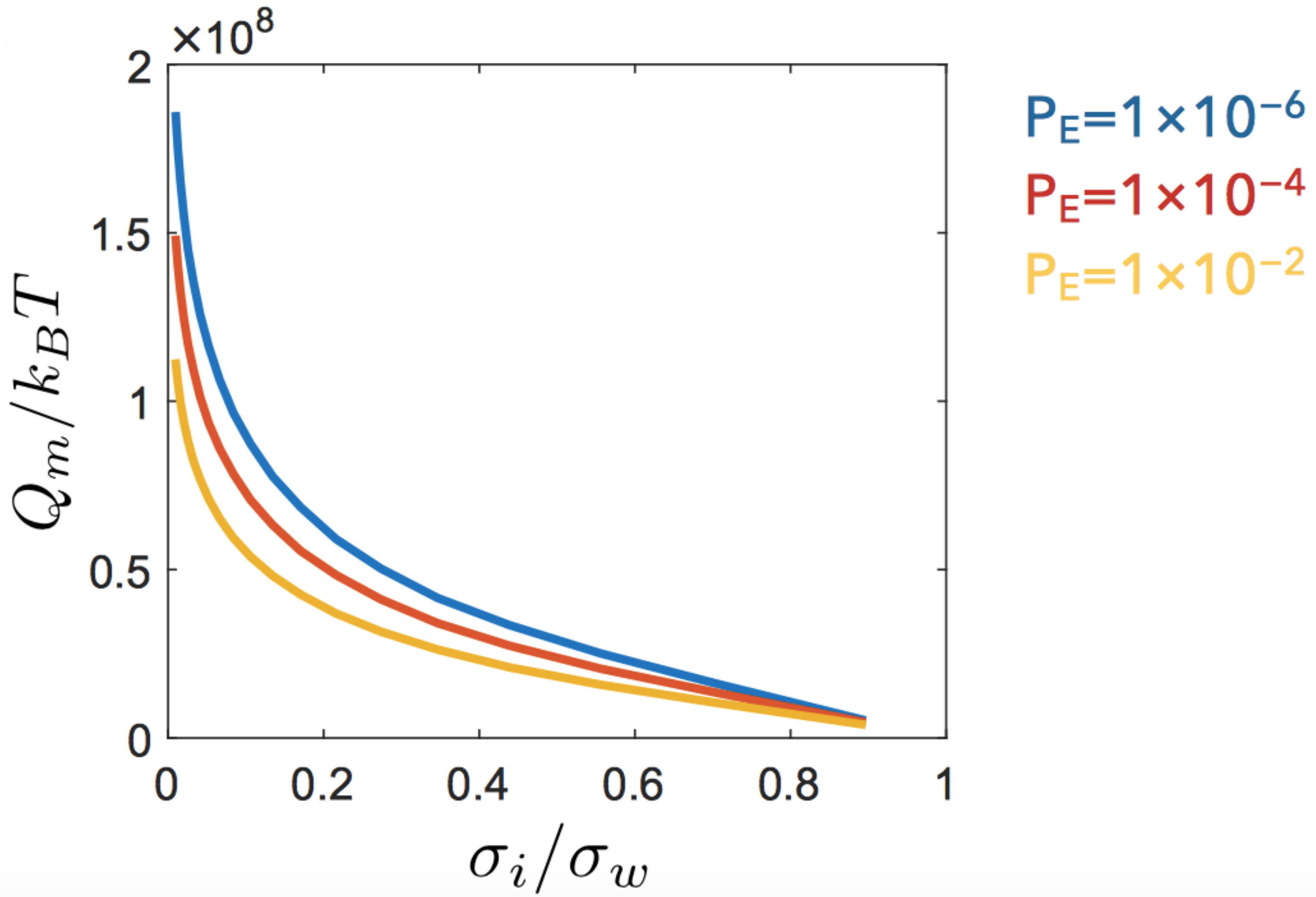
MINIMUM ENERGY REQUIRED TO PRESERVE A MEMORY OVER A FIXED TIME WITH A GIVEN ERROR PROBABILITY

$$Q_m = -NT\Delta S = \frac{t}{t_R} k_B T \ln\left(\frac{\sqrt{\left(\sigma_w^2 + e^{-\frac{t_R}{\tau_w}}\left(\sigma_i^2 - \sigma_w^2\right)\right)}}{\sigma_i}\right)$$

Where: σ_i is the initial probability density width

σ_w is the equilibrium (inside one well) probability density width

3 Determine the minimum energy for keeping the memory



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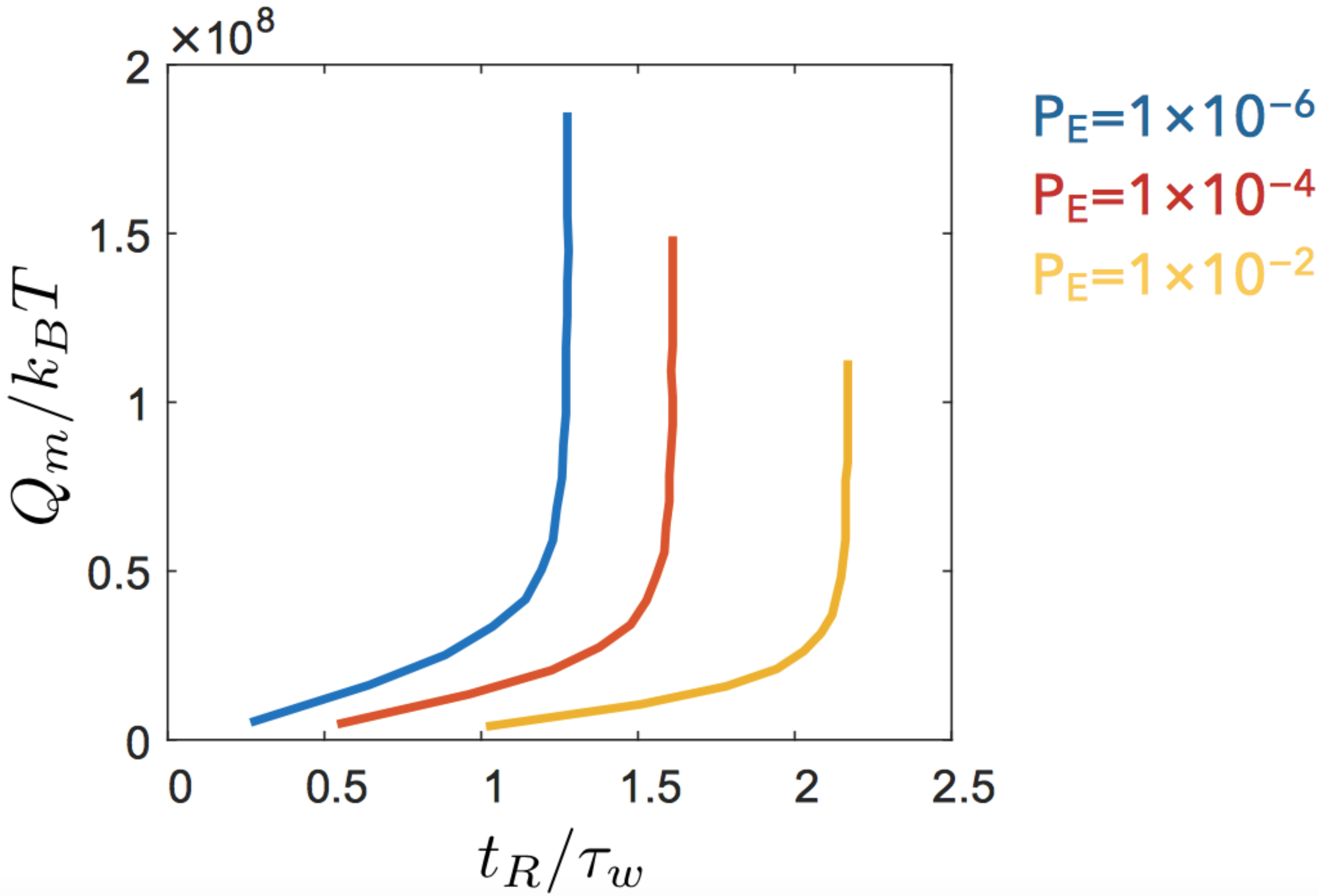
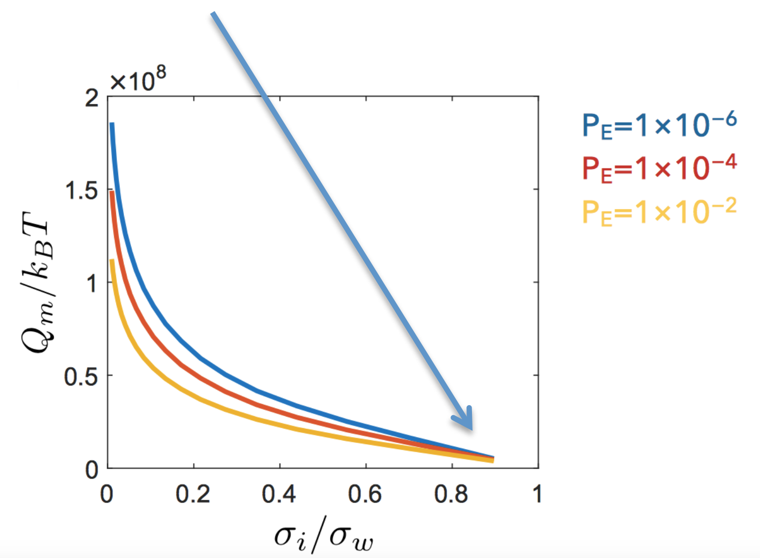


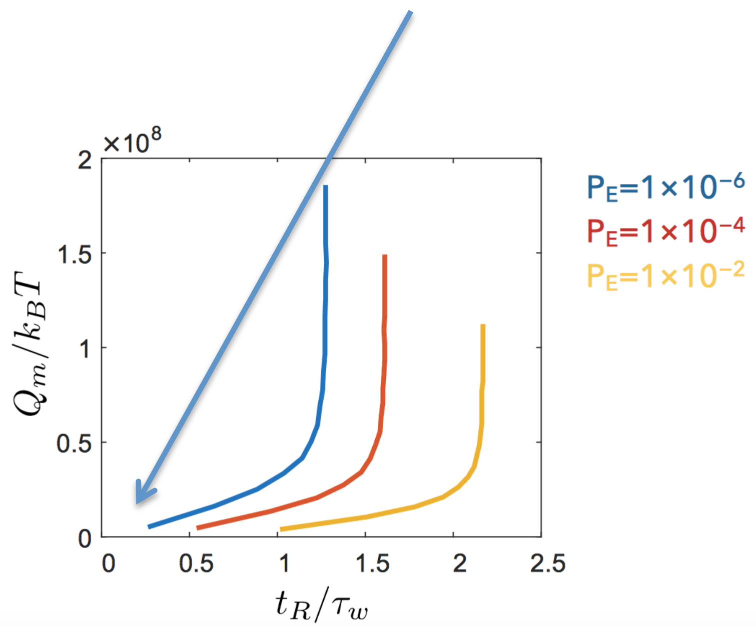
4 Considerations

The good news!

We can preserve a memory for a given time with a given error probability while spending an arbitrarily little amount of energy.

This is accomplished if the refresh procedure is performed arbitrarily often or arbitrarily close to thermal equilibrium.





4 Considerations

The bad news!

If we consider the relation:

$$P_E = 1 - \left[1 - P_0\left(t_R\right)\right]^{\frac{t}{t_R}}$$

we have:

$$t = t_R \ln(1 - P_E)/\ln(1 - P_0)$$

Once we set P_E and select a finite t_R, we can make **t** as large as we want by properly selecting P_0 small enough.

However P_0 cannot be made arbitrarily small without spending a finite amount of energy.
This can be seen in terms of the width σ_0 of the initial distribution.
If we want to make $\sigma_0 = 0$, we need to perform an operation that changes the system entropy form a given σ_i to $\sigma_f = \sigma_0$.
As we have seen, the associated change in entropy is provided by

$$\Delta S = k_B \ln\left(\frac{\sigma_i}{\sigma_f}\right)$$

Thus when $\sigma_f = \sigma_0 \rightarrow 0$ the entropy change tends to diverge and so does the energy required to perform this operation.

4 Considerations

Another way to look at this problem is to consider the **Heisenberg Indetermination principle** that prevents the arbitrary confinement of the probability density, without spending an infinite amount of energy: the uncertainty on the impulse diverges when the uncertainty on the position shrinks.

In the best scenario we have: $\sigma_x \sigma_p = \frac{\hbar}{2}$

If the memory setting operation is performed at thermal equilibrium

we have $\sigma_p = m\sqrt{\langle v^2 \rangle - \langle v \rangle^2} = \sqrt{m k_B T}$

and thus $\sigma_x = \frac{\hbar}{2\sqrt{m k_B T}}$

Thus it exists a σ_{MIN} and it has to be $\sigma_i \geq \sigma_{iMin} = \frac{\hbar}{2\sqrt{m k_B T}}$

4 Considerations

Example:

If we assume the distance between the two wells $\mathbf{x}_m$ = 1 nm and a refresh period $\mathbf{t}_R$ = 6.6 10^{-3} s, we have that the minimum σ_i = 9. 6 10^{-20} m.

For $\boldsymbol{P}_E$ = 1 10^{-6} then the maximum value for **t** is approximately 2 years.
For $\boldsymbol{P}_E$ = 1 10^{-4} then the maximum time **t** is approximately 200 years.

4 Considerations

The existence of a σ_{MIN} implies that the probability of error P_0 cannot be arbitrarily small

and, thus $t = t_R \ln(1 - P_E) / \ln(1 - P_0)$ cannot be arbitrarily large

For any P_E we select we have an associate maximum for the memory duration **t**

The good news: You can keep your memory by spending 0 energy

The bad news: A memory cannot last forever

Conclusion

Take home message

You can preserve your memory only for a limited amount of time.
Within this limit, if you do things carefully enough, you do not need to spend any energy.

More on:

The cost of remembering one bit of information
Davide Chiuchiù, Miquel López-Suárez, Igor Neri, Maria Cristina Diamantini, Luca Gammaitoni.
Physical Review A 97 (5), 052108, 2018